



Name..... MARKING SCHEME ADM Number:.....

School:..... URANGA JOINT Candidate's Signature..... [Signature]

121
 Mathematics Alt.A
 FORM THREE.
 OCTOBER 2ND TERM 3 2024.
 2 ½ Hours.

URANGA MATHEMATICS ASSOCIATION-2024.
 Kenya Certificate of Secondary Education
 MATHEMATICS 121
 FORM THREE
 TIME: 2 ½ HOURS

INSTRUCTIONS TO CANDIDATES:

- Write your name, school, admission number and sign in the spaces provided above.
- This paper contains **TWO** sections: Section I and Section II.
- Answer **ALL** the questions in Section I and **FIVE** questions from section II.
- All answers and working **MUST** be written on the question paper in the spaces provided below each question.
- Marks may be given for correct working even if the answer is wrong.
- Non-programmable silent electronic calculators and KNEC Mathematical tables may be used, except where stated otherwise.

FOR EXAMINERS USE ONLY

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

SECTION II

17	18	19	20	21	22	23	24	Total

**Grand
 Total**

This paper consists of 15 printed pages. Candidates should check to ascertain that all pages are printed as indicated and that no questions are missing.

1. Simplify leaving your answer in the form of $a\sqrt{b} + c$ where a, b and c are integers. (3mks)

$$\frac{\sqrt{48}}{\sqrt{5} + \sqrt{3}}$$

$$\frac{\sqrt{48}}{(\sqrt{5} + \sqrt{3})} \frac{\sqrt{5} - \sqrt{3}}{(\sqrt{5} - \sqrt{3})} \quad \text{u/}$$

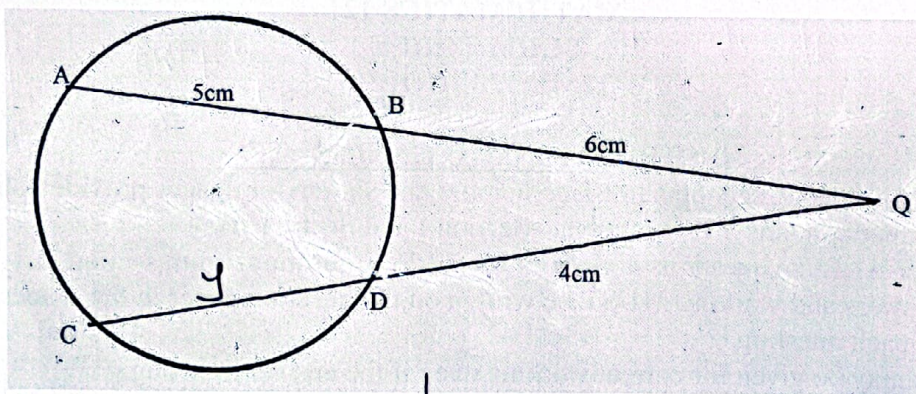
$$\frac{\sqrt{240} - \sqrt{144}}{5 - 3}$$

$$\frac{4\sqrt{15} - 12}{2} \quad \text{u/}$$

$$2\sqrt{15} - 6 \quad \text{A/}$$

(03)

2. Chords AB and CD in the figure below intersect externally at Q. if AB = 5cm BQ = 6cm and DQ = 4cm, calculate the length of chord CD. (3 Marks)



$$(5+6)6 = (4+y)4 \quad \text{u/}$$

$$66 = 16 + 4y$$

$$\frac{50}{4} = \frac{4y}{4} \quad \text{u/}$$

$$CD = 12.5 \text{ cm.} \quad \text{A/}$$

(03)

3. Evaluate, without using mathematical tables or the calculator, the expression. (3 marks)

$$2 \log_{10} 5 - \frac{1}{2} \log_{10} 16 + 2 \log_{10} 40$$

$$\log 5^2 - \log \sqrt{16} + \log 40^2$$

$$\log \left(\frac{25}{4} \times 40^2 \right) \quad \text{u/}$$

$$\log 10000 \quad \text{A/}$$

$$= 4 \quad \text{B/}$$

(03)

4. A rectangular card measures 5.0cm by 3cm. Find the percentage relative error in the area of the card correct to 3dp. (3mks)

$$\text{Max Area} = 5.05 \times 3.5 = 17.675$$

$$\text{Min Area} = 4.95 \times 2.5 = 12.375$$

$$\text{Actual Area} = 5 \times 3 = 15$$

$$\text{A.E.} \Rightarrow \frac{17.675 - 12.375}{2} = 2.65$$

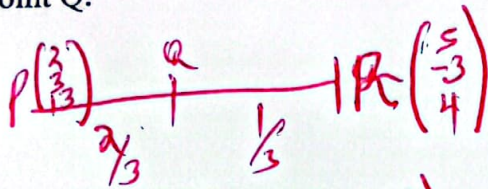
$$\text{RE} \Rightarrow \frac{2.65}{15} \times 100$$

$$\Rightarrow 17.67\%$$

03

$$\frac{\text{A.E.}}{\text{Actual Area}} = \frac{2.65}{15} = 0.1767 \times 100 = 17.67\%$$

5. The points P, Q and R lie on a straight line. The position vectors of P and R are $2\mathbf{i} + 3\mathbf{j} + 13\mathbf{k}$ and $5\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ respectively; Q divides PR internally in the ratio 2: 1. Find the co-ordinates of point Q. (3 marks)



$$\text{OR} \Rightarrow \frac{2}{3} \begin{pmatrix} 2 \\ 3 \\ 13 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} \frac{2}{3} \\ \frac{3}{3} \\ \frac{13}{3} \end{pmatrix} + \begin{pmatrix} \frac{10}{3} \\ -\frac{6}{3} \\ \frac{8}{3} \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \\ 7 \end{pmatrix}$$

$$Q(4, -1, 7)$$

6. A and B are two matrices. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ find B given that $A^2 = A + B$. (3 marks)

$$A^2 \Rightarrow \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 9 & 8 \\ 16 & 17 \end{pmatrix}$$

$$A + B \Rightarrow A^2$$

$$\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 9 & 8 \\ 16 & 17 \end{pmatrix}$$

$$B = \begin{pmatrix} 8 & 6 \\ 12 & 14 \end{pmatrix}$$

03

7. A customer deposited sh 14000 in a saving account. Find the accumulated interest, correct to 3 significant figures, after two years if interest was paid at 12% p.a compounded quarterly (4mks)

$$A = P \left(1 + \frac{r}{100} \times \frac{1}{4}\right)^{n \times 4}$$

$$14000 \left(1 + \frac{12}{100} \times \frac{1}{4}\right)^8$$

$$\underline{17,700}$$

(03)

8. Use completing the square method to solve the equation

$$4 - 3x - 2x^2 = 0$$

$$P \propto Q^3 + \frac{1}{Q^2}$$

$$P = kQ^3 + \frac{1}{Q^2} \quad B$$

$$259 = 27k + \frac{1}{9}$$

$$2331 = 243k + h - 0$$

(3marks)

$$108 = 8k + h$$

$$432 = 32k + h$$

$$\frac{2331}{-1899} = \frac{-211k}{-1899}$$

$$k = 9$$

$$h = 144$$

$$P = 9Q^3 + \frac{144}{Q^2}$$

$$P = 9(6^3) + \frac{144}{6^2}$$

$$P = 1948 \quad B1$$

off

9. A quantity P varies partly as the cube of Q, and partly varies inversely as the square of Q. When Q = 2, P = 108 and when Q = 3, P = 259. Find the value of P when Q = 6 (4marks)

$$P \propto Q^3 + \frac{1}{Q^2}$$

$$P = kQ^3 + \frac{1}{Q^2}$$

$$259 = 27k + \frac{1}{9} \quad (1)$$

$$2331 = 243k + h - 0$$

$$108 = 8k + \frac{1}{8}, \quad 432 = 32k + h - 0$$

$$\frac{2331}{432} = \frac{243k + h}{32k + h}$$

$$1899 = 211k$$

$$9 = k, \quad h = 144$$

$$P = 9Q^3 + \frac{144}{Q^2}$$

$$P = 6^3(9) + \frac{144}{6^2}$$

$$\Rightarrow \underline{1948}$$

10. Make t the subject of the formula in $M = \frac{2}{3P} \frac{F}{2K-t}$ (3 marks)

$$\left(\frac{3PM}{2}\right)^2 = \left(\frac{F}{2K-t}\right)^2$$

$$\frac{9P^2M^2}{4} = \frac{F}{2K-t}$$

$$18P^2MK - 9P^2Mt = 4F$$

$$\frac{18P^2MK - 4F}{9P^2M} = \frac{9P^2Mt}{9P^2M}$$

$$t \Rightarrow \frac{18P^2MK - 4F}{9P^2M}$$

11. The roots of a quadratic equation are $\frac{2}{3}$ and $-\frac{1}{5}$ respectively. Express the equation in the form

$ax^2 + bx + c = 0$ where a, b and c are integral values. (3 marks)

$$x = \frac{2}{3}, x = -\frac{1}{5}$$

$$(3x-2)(5x+1) = 0$$

$$15x^2 + 3x - 10x - 2 = 0$$

$$15x^2 - 7x - 2 = 0$$

12. Points A and B have the coordinates (3, -2, 1) and (4, 1, -3) respectively. Express the column vector \vec{AB} in terms of i, j and k . (3 marks)

$$\vec{AB} = B - A$$

$$\begin{pmatrix} 4 \\ 1 \\ -3 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix}$$

$$\vec{AB} \Rightarrow i + 3j - 4k$$

13. Grade A rice costing Ksh 250 per kg is mixed with Grade B rice costing ksh. 180 per kg. In what ratio must we mix Grade A to Grade B so that a profit of 20% is realized by selling the blend at Ksh 240 per kg. (3 marks)

$$240 = 120\%$$

$$7. = 100\%$$

$$\frac{240 \times 100}{120}$$

$$200 \Rightarrow 200$$

$$\begin{array}{ccc} 250 & 180 & \\ & \swarrow & \searrow \\ & 200 & \\ & \swarrow & \searrow \\ 20 & & 50 \end{array}$$

$$2:5$$

14. Expand $(3 + x)^5$ in ascending powers of x up to the term in x^3 . Hence use your expansion to evaluate $(3.2)^5$ (4 mark)

3	243	81	27	9
x	1	x	x^2	x^3
	1	5	10	10

$$243 + 405x + 270x^2 + 90x^3$$

$3.2 = 3 + x$
 $x = 0.2$

$$243 + 405(0.2) + 270(0.2)^2 + 90(0.2)^3$$

$$\Rightarrow 335.52$$

04

15. The sixth term of an arithmetic progression is 27 and the tenth term is 43. Find the 16th term. (3 marks)

$$a + 5d = 27$$

$$a + 9d = 43$$

$$-4d = -16$$

$$d = 4, a = 7$$

$$a + 15d =$$

$$7 + 4(15)$$

$$67$$

16. Solve the equation

$$\sin(x + 10)^\circ = -0.5 \text{ for } 0^\circ \leq x \leq 360$$

$$x + 10 = \sin^{-1}(-0.5)$$

$$\Rightarrow -30$$

$$x + 10 = 210, 330$$

$$x \Rightarrow 200, 320$$

03

(3mks)

SECTION II (50 MARKS)

Answer FIVE questions ONLY from this section

17. The table below shows income tax rates per annum.

Taxable pay K£	Rate of tax sh per K£
1-3420	2
343-6840	3
685-10260	4
1027-13680	5
1369-17100	6
Over 17100	7

A civil servant earns a monthly salary of sh 20 000. He also earn medical allowance of ksh 2840 and travelling allowance of Ksh 5000. He is provided with a house at a nominal rent of sh 700 per month.

a) Calculate the civil servant's monthly taxable income.

(3mks)

$$TI = (BS + \frac{15}{100} BS + AL) - NFR$$

$$20,000 + \frac{15}{100} \times 20,000 + 2840 + 5000 - 700$$

$$1400 \Rightarrow 30140 \text{ K£ } 1507$$

03

b) If the employee is entitled to a tax relief of sh 7200 per annum. Determine the civil servant's net tax per month in K£.

G.T. = 78728 5mks

$$PATE \frac{78728 - 7200}{12}$$

$$\Rightarrow 5960.70$$

05

- 1- 3420 x 2 = 6840
- 2- 3420 x 3 = 10260
- 3- 3420 x 4 = 13680
- 4- 3420 x 5 = 17100
- 5- 3420 x 6 = 20520
- 6- 3420 x 7 = 23940
- 7- 3420 x 8 = 27360

$$1507 \times 12 = 18084$$

$$18084 - 7200 = 10884$$

c) determine the servant's net monthly income in Ksh. [2mks]

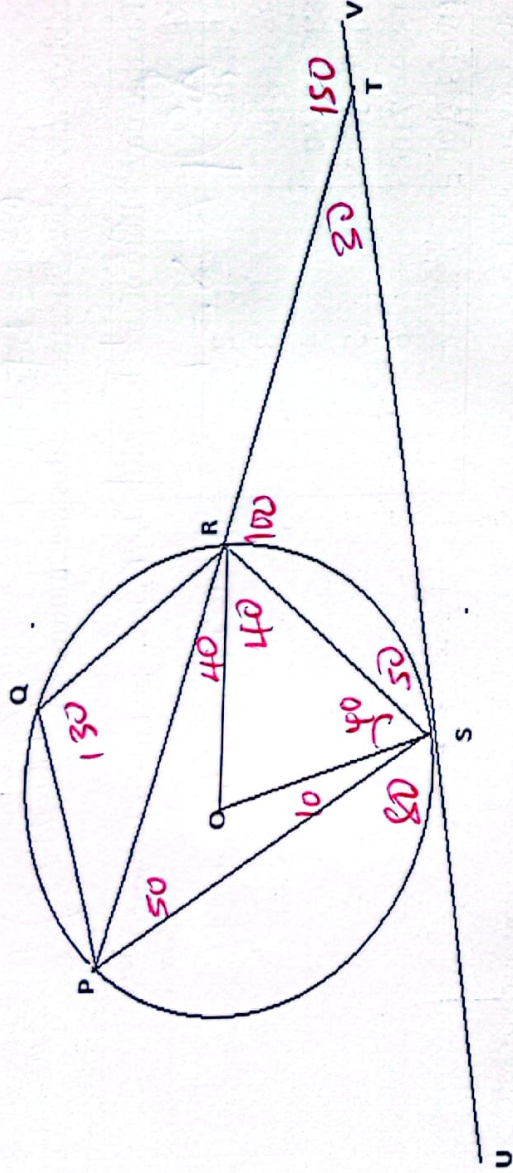
$$(30140 - 5960.70) \times 12$$

$$24879.30$$

02

10

18. In the diagram shown below O is the centre of the circle, angle $\text{RTV} = 150^\circ$, and angle $\text{RST} = 50^\circ$.



a) Calculate the size of

i. $\angle \text{ORS}$

40° $\left\{ \begin{array}{l} \text{isosceles triangle} \\ \text{base angles are equal} \end{array} \right.$ (2mks)

ii. $\angle \text{USP}$

80° $\left\{ \begin{array}{l} \text{Angle by a straight line} \\ \text{add up to } 180^\circ \end{array} \right.$ (2mks)

iii. $\angle \text{PQR}$

130° $\left\{ \begin{array}{l} \text{Cyclic quadrilateral} \\ \text{opposite angles add up to } 180^\circ \end{array} \right.$ (2mks)

b) Given that $\text{RT} = 7\text{cm}$ and $\text{ST} = 9\text{cm}$, calculate to 3.s.f

i. The length of line PR

$$\begin{aligned} 9^2 &= (7+y)^2 \\ 81 &= 49 + 14y \\ y &= 4.57 \end{aligned}$$

(2mks)

ii. The radius of the circle

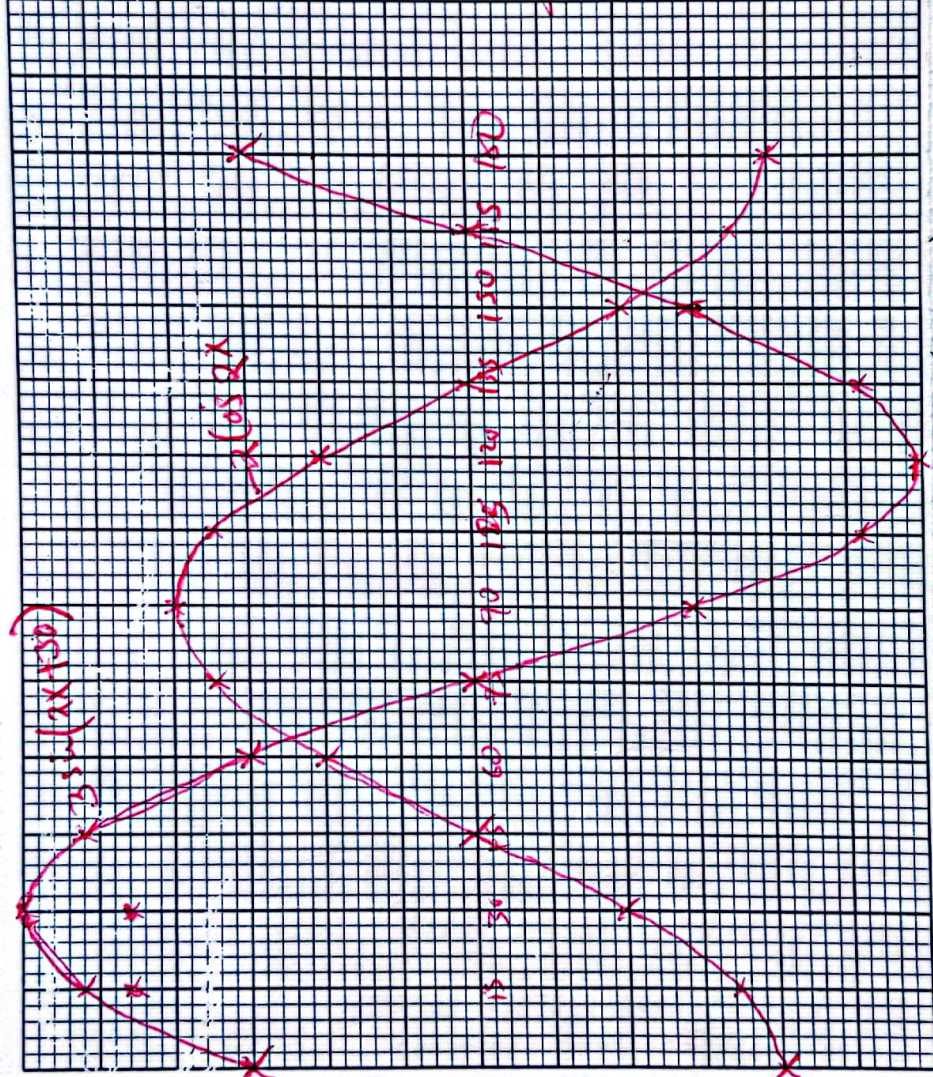
$$\begin{aligned} \frac{4.57}{\sin 50} &= 2R \\ R &= 2.98 \end{aligned}$$

(3mks)

19. (a) Complete the table below correct to 2 decimal places. (2 marks)

x°	0	15	30	45	60	75	90	105	120	135	150	165	180
$-2 \cos 2x$	-2	-1.73	-1.00	0.00	1.00	1.73	2.00	1.73	1.00	0.00	-1.00	-1.73	-2.00
$3 \sin(2x + 30^\circ)$	1.50	2.60	3.00	2.60	1.50	0	-1.50	-2.60	-3.00	-2.60	-1.50	0.00	1.50

(b) Using a scale of 1 cm for 15° on the x-axis and 2 cm for 1 unit on the y-axis, and on the same axes, draw the graphs of $y = 2 \cos 2x$ and $y = 3 \sin(2x + 30^\circ)$ for $0^\circ \leq x \leq 180^\circ$.



(5 mark)

82
51
51
5
05

(c) Using the graph in (b) above, solve the equation $3 \sin(2x + 30^\circ) + 2 \cos 2x = 0$. (2 marks)

$x = 153$ or 63

02

(1 mark)

(d) Find the range of values of x for which $3 \sin(2x + 30^\circ) \leq -2 \cos 2x$.

$63 \leq x \leq 153$

01

20a). The first term of an arithmetic progression is 2. The sum of the first 8 terms of the AP series is 240.

(i). Find the common difference of the AP.

(2 marks)

$$S_n = \frac{n}{2} (2a + (n-1)d)$$

$$\frac{8}{2} (2 \times 2 + (8-1)d) = 240$$

$$4(4 + 7d) = 240$$

$$4 + 7d = 60$$

$$7d = 56$$

$$d = 8$$

(ii). Given that the sum of the first n terms of the AP is 1560. Find n

(3 marks)

$$S_n = \frac{n}{2} (2a + (n-1)d)$$

$$\frac{n}{2} (2 \times 2 + (n-1)8) = 1560$$

$$n(4 + 8n - 8) = 3120$$

$$8n^2 - 4n = 3120$$

$$2n^2 - n - 780 = 0$$

$$2n^2 - 40n + 39n - 780 = 0$$

$$2n(n-2) + 39(n-20) = 0$$

$$2n = -39 \text{ or } n-2 = 0$$

$$n = -19.5 \text{ or } 20, \quad n = 20$$

b) The 3rd, 5th and 8th terms of another AP form the first three terms of a GP. If the common difference of the AP is 3. Find:

(i). the first term of the GP.

(3 marks)

$$a + 2d, \quad a + 4d, \quad a + 7d$$

$$\frac{a + 4d}{a + 2d} = \frac{a + 7d}{a + 4d}$$

$$a^2 + 12a + 12a + 14d = a^2 + 6a + 21d + 26$$

$$24a - 27a = 126 - 14d$$

$$-3a = -18$$

$$a = 6$$

$$a \Rightarrow 6 + 2(3) = 12$$

(ii). the sum of the first 9 terms of the GP to 4 significant figures.

(2 marks)

$$r = \frac{27}{18} = 1.5$$

$$S_n \Rightarrow a \left(\frac{r^n - 1}{r - 1} \right)$$

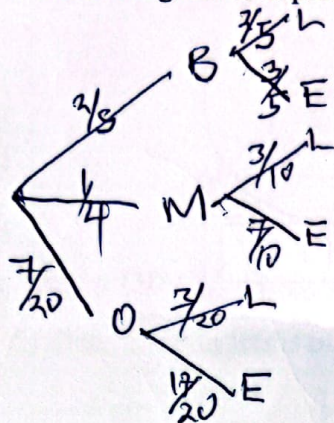
$$12 \left(\frac{1.5^9 - 1}{1.5 - 1} \right)$$

$$S_n \Rightarrow 898.6$$

21. The probability that Zora goes to school by boda boda is $\frac{2}{5}$ and by a matatu is $\frac{1}{4}$. If she uses a boda boda, the probability that she will be late is $\frac{2}{5}$ and $\frac{3}{10}$ if she uses a matatu. If she uses other means of transport the probability of being late is $\frac{3}{20}$

a) Draw a tree diagram to represent this information.

(3 marks)



By By

02

b) Find the probability that she will be late for school

(3 marks)

$$\left(\frac{2}{5} \times \frac{2}{5}\right) + \left(\frac{1}{4} \times \frac{3}{10}\right) + \left(\frac{7}{20} \times \frac{3}{20}\right)$$

W
W

05

$$\frac{23}{80} \quad \star$$

c) Find the probability that she will be late for school if she does not use a matatu (3 marks)

$$\left(\frac{2}{5} \times \frac{2}{5}\right) + \left(\frac{7}{20} \times \frac{3}{20}\right)$$

$$\frac{17}{80} \quad \star$$

08

d) Find the probability that she will be late for school

(2 marks)

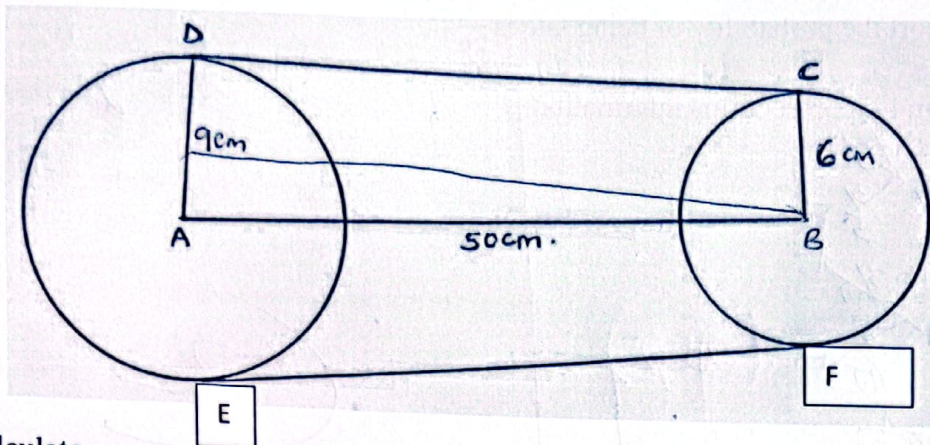
$$1 - P(L)$$

$$1 - \frac{23}{80}$$

$$\frac{57}{80} \quad \star$$

08

22. The figure below shows two pulleys with centres A and B of radii 9 cm and 6 cm respectively. C and D are contact points of the belt with the pulleys. The distance between the centres of the two pulleys is 50 cm. A belt is tied around the two pulleys as shown.



Calculate,

(e) Length DC (2 marks)

$$\begin{array}{|l} \left. \begin{array}{l} \text{Right-angled triangle with hypotenuse } 50 \text{ and base } 3 \\ \Rightarrow \sqrt{50^2 - 3^2} \\ 49.91 \text{ cm} \end{array} \right\} \end{array}$$

02

(f) The length of arc DE (3 marks)

$$\begin{array}{|l} \left. \begin{array}{l} \text{Right-angled triangle with hypotenuse } 50 \text{ and base } 3 \\ \cos \theta = \frac{3}{50} \\ \theta = 173.12^\circ \\ 360 - 173.12 \end{array} \right\} \end{array}$$

$$\frac{186.88 \times 2 \times \frac{22}{7} \times 9}{360} = 29.37$$

03

(g) The length of arc CF (3 marks)

$$\frac{173.12 \times 2 \times \frac{22}{7} \times 6}{360} \Rightarrow 18.14$$

03

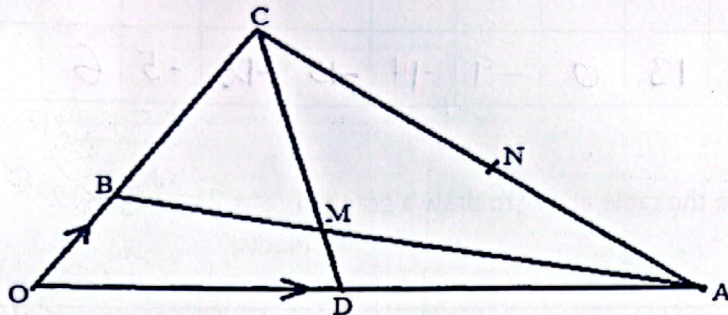
(h) The total length of the belt. (2 marks)

$$18.14 + 29.37 + 49.91 + 49.91 \Rightarrow 147.33$$

02

10

23. In the figure below $\vec{OB} = \underline{b}$, $\vec{OC} = 3\vec{OB}$ and $\vec{OA} = \underline{a}$



(a) Given that $\vec{OD} = \frac{1}{3} \vec{OA}$ and $\vec{AN} = \frac{1}{2} \vec{AC}$, CD and AB meet at M .

(i) Determine in terms of \underline{a} and \underline{b} vector \vec{AB}

(1 mark)

$$-\underline{a} + \underline{b} \parallel \underline{b} - \underline{a} \quad \text{B}$$

(ii) Determine in terms of \underline{a} and \underline{b} vector \vec{CD}

(1 mark)

$$\frac{1}{3}\underline{a} - 3\underline{b} \parallel \underline{b} - \frac{1}{3}\underline{a} \quad \text{B}$$

$$-3\underline{b} + \frac{1}{3}\underline{a}$$

(b) Given that $\vec{CM} = k\vec{CD}$ and $\vec{AM} = h\vec{AB}$, determine the values of the scalars k and h .

(5 marks)

$$\vec{CM} = k(\frac{1}{3}\underline{a} - 3\underline{b})$$

$$\frac{1}{3}k\underline{a} - 3k\underline{b} \quad \text{m}$$

$$\vec{CM} = (\underline{a} - 3\underline{b}) + h(-\underline{a} + \underline{b})$$

$$\underline{a} - 3\underline{b} - h\underline{a} + h\underline{b}$$

$$\underline{a}(1-h) + \underline{b}(-3+h) \quad \text{m}$$

$$\frac{1}{3}k\underline{a} = \underline{a}(1-h)$$

$$\frac{1}{3}k = 1-h \quad \text{(ii)}$$

$$\underline{b}(h-3) = -3k\underline{b}$$

$$h-3 = -3k$$

$$h-3 = -3(3-3h)$$

$$h-3 = -9+9h$$

$$6 = 8h$$

$$h = \frac{3}{4}$$

$$\frac{1}{3}k = 1 - \frac{3}{4}$$

$$k = \frac{3}{4}$$

05

(c) Show the O , M and N are collinear.

(3 marks)

$$\vec{OM} = 3\underline{b} + \frac{3}{4}(\frac{1}{3}\underline{a} - 3\underline{b})$$

$$\frac{1}{4}\underline{a} + \frac{3}{4}\underline{b}$$

$$\vec{ON} = 3\underline{b} + \frac{1}{2}(\underline{a} - 3\underline{b})$$

$$\frac{1}{2}\underline{a} + \frac{3}{2}\underline{b}$$

$\vec{ON} = 2\vec{OM}$ Common point O , thus O, M, N are collinear.

03

24. Complete the table given below for the equation $y = 2x^2 + 5x - 12$ for $-8 \leq x \leq 4$ by filling the blank space in the table below.

-5	-4	-3	-2	-1	0	1	2
13	0	-9	-14	-15	-12	-5	6

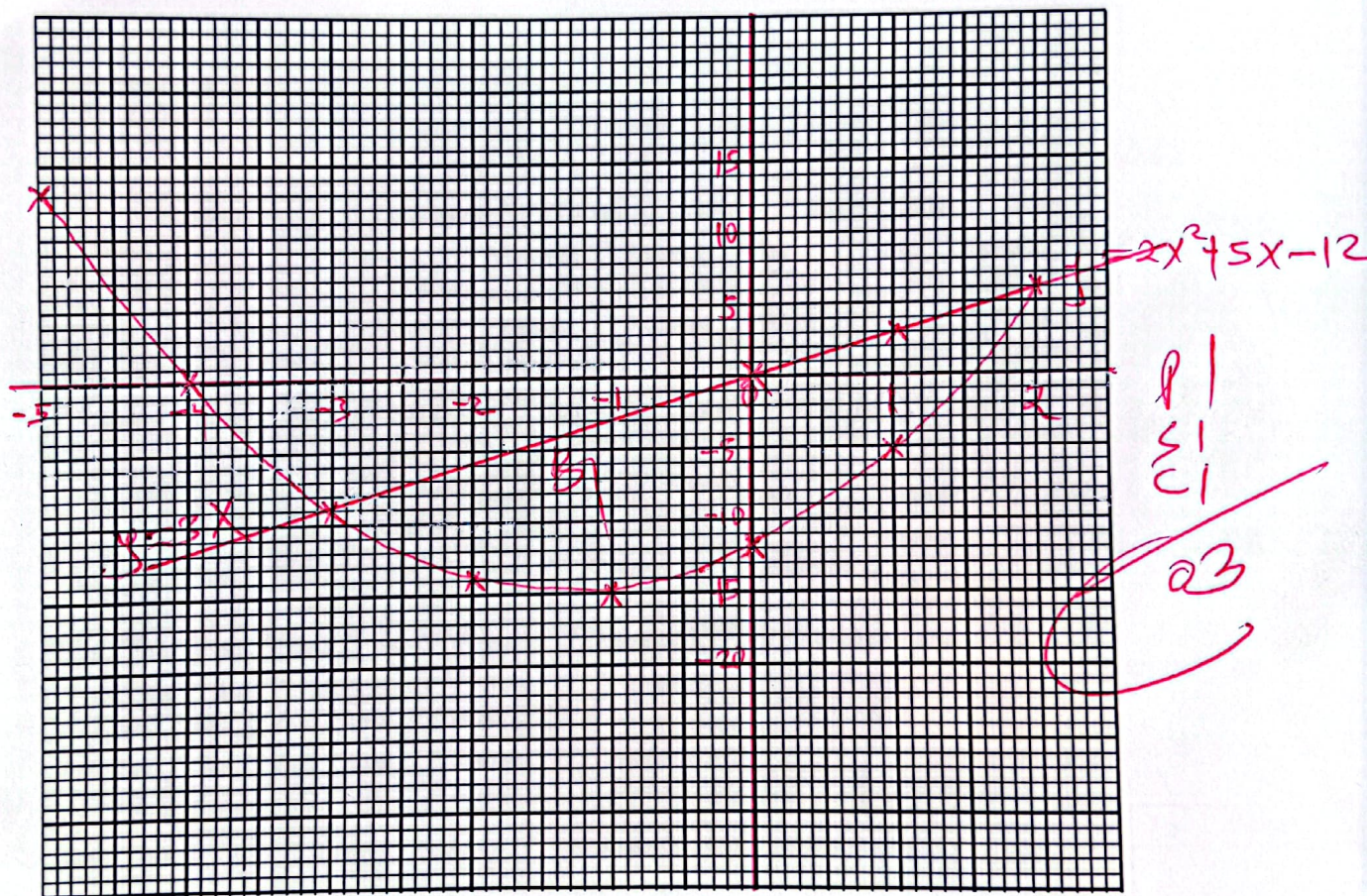
B2

02

(2 marks)

a) Use the values from the table above to draw a graph of $y = 2x^2 + 5x - 12$

(3 marks)



b) Use the graph drawn above to solve for x in the equations below

$$2x^2 + 5x - 12 = 0$$

$$x = -4 \text{ or } x = 1.6 \text{ B1}$$

$$x^2 + x - 6 = 0$$

$$\begin{array}{l} y = 2x^2 + 5x - 12 \quad \left. \begin{array}{l} -1 \\ -1 \end{array} \right\} \\ 0 = x^2 + x - 6 \quad \left. \begin{array}{l} -1 \\ 2 \end{array} \right\} \end{array} \quad \text{m1}$$

$$\begin{array}{r} y = 2x^2 + 5x - 12 \\ 0 = 2x^2 + 2x - 12 \\ \hline y = 3x \quad \text{m1} \end{array}$$

$$\begin{array}{c|c|c|c} x & 0 & 1 & 2 \\ \hline y & 0 & 3 & 6 \end{array}$$

$$x = -3 \text{ or } 2 \text{ B1}$$

1mk
(2 mark)

0

(3 marks)

04