Name: MARKING GUIDE ADM Number: F4P2
School: URANGA MATHS Candidate's Signature................................

121/2
Mathematics Alt.A
FORM FOUR.
JULY 3RD 2024.
2 ½ Hours.

URANGA MATHEMATICS ASSOCIATION-2024.

Kenya Certificate of Secondary Education
MATHEMATICS 121/2
FORM FOUR
TIME: 2 ½ HOURS

INSTRUCTIONS TO CANDIDATES:

- Write your name, school, admission number and sign in the spaces provided above.
- This paper contains **TWO** sections: Section **I** and Section **II**.
- Answer **ALL** the questions in Section **I** and **FIVE** questions from section **II**.
- All answers and working **MUST** be written on the question paper in the spaces provided below each question.
- Marks may be given for correct working even if the answer is wrong.
- Non-programmable silent electronic calculators and KNEC Mathematical tables may be used, except where stated otherwise.

FOR EXAMINERS USE ONLY

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

SECTION II

17	18	19	20	21	22	23	24	Total

Grand Total

This paper consists of 15 printed pages. Candidates should check to ascertain that all pages are printed as indicated and that no questions are missing.

SECTION I (50Marks)

Answer all questions in this section in the spaces provided.

1. A dealer has two types of grades of tea, A and B. Grade A costs shs. 140 per kg while grade B costs shs.160 per kg. If the dealer mixes A and B ratio 3:5 to make a brand of tea which he sells at shs.180 per kg, calculate his percentage profit (3 marks)

Cost per kg $= \frac{(140 \times 3) + (160 \times 5)}{8}$ — M1

$= $ sh. 152.50.

% Profit $= \left(\frac{180 - 152.50}{152.50}\right) \times 100\%$ — M1

$= 18.03\%$ — A1

03.

2. Solve for x in the equation $2sin^2x + cosx = -1\ for 0^0 \leq x < 360^0$ (4 marks)

$2(1 - \cos^2 x) + \cos x = -1$ — M1

Let $\cos x = y$.

$2(1 - y^2) + y = -1$

$2y^2 - y - 3 = 0$.

$2y^2 + 2y - 3y - 3 = 0$ — M1

$2y(y+1) - 3(y+1) = 0$.

$(2y-3)(y+1) = 0$.

$y = 1\frac{1}{2}$ or -1 — A1

$\cos x = -1$

$x = \cos^{-1}(-1)$

$= 180°$. — B1

04.

3. Make c the subject of the formula (3 marks)

$(x)^2 = \left(\frac{\sqrt{(c^2+df)}}{cy}\right)^2$

$x^2 = \frac{c^2 + df}{c^2y^2}$ — M1

$c^2x^2y^2 = c^2 + df$

$c^2x^2y^2 - c^2 = df$

$c^2(x^2y^2 - 1) = df$ — M1

$c^2 = \frac{df}{x^2y^2-1}$

$c = \pm\sqrt{\frac{df}{x^2y^2-1}}$ — A1

03

Note: Accept without ±.

4. A quadratic curve cuts the x – axis at points (-2, 0) and (3, 0). Find the equation of this curve in the form $ax^2 + by = c$ where a, b and c are integers. (3 marks)

$(x+2)(x-3) = 0$ — M1

$x^2 - 3x + 2x - 6 = 0$ — M1

$x^2 - x = 6$ — A1

03

5. The equation of a circle is $2x^2 + 2y^2 - 16x + 12y - 22 = 0$. Determine the centre and radius of the circle. (3 marks)

$x^2 + y^2 - 8x + 6y - 11 = 0$

$x^2 - 8x + y^2 + 6y = 11$ — M1

$x^2 - 8x + 16 + y^2 + 6y + 9 = 11 + 16 + 9$

$(x-4)^2 + (y+3)^2 = 6^2$ — M1

$C(4, -3)$, $r = 6$ units — A1

03.

6. Expand $(2 + 3x)^6$ up to the term in x^3. Hence use your expansion to estimate $(2.09)^6$ (3 marks)

$\Rightarrow 2^6(3x)^0 + 2^5(3x)^1 + 2^4(3x)^2 + 2^3(3x)^3$

1 6 15 20

$64 + 576x + 2160x^2 + 4320x^3$ — B1

$2 + 3x = 2.09$

$x = 0.03$

$= 64 + 576(0.03) + 2160(0.03)^2 + 4320(0.03)^3$ — M1

$= 83.34$ — A1

03

7. The diameter of a spherical ball is measured as 14cm, correct to the nearest centimeter. Determine, to 2 decimal places, the percentage error in calculating the volume of the ball. (3 marks)

Max Volume $= \frac{4}{3}\pi \times 14.5^3 = 4064\frac{5}{6}\pi$

Actual volume $= \frac{4}{3}\pi \times 14^3 = 3658\frac{2}{3}\pi$

Min volume $= \frac{4}{3}\pi \times 13.5^3 = 3280\frac{1}{2}\pi$

$$AE = \frac{4064\frac{5}{6} - 3280\frac{1}{2}}{2} \quad \text{— M1}$$

$$= 392\frac{1}{6}$$

$$\%E = \frac{392\frac{1}{6}}{3658\frac{2}{3}} \times 100\% \quad \text{— M1}$$

$$= 10.72\% \quad \text{— A1}$$

03

OR

$$\left(\frac{0.5 + 0.5 + 0.5}{14}\right) \times 100 \quad \text{M1 M1}$$

$$= 10.72\% \quad \text{A1}$$

8. Given that O is the origin and **OA** = 2**i** + 2**j** – 4**k** and **OB** = 6**i** + 10**j** + 2**k**. If R divides AB externally in the ratio 3:1, find **OR** in terms of **i**, **j** and **k**. (3 marks)

$$\underset{\sim}{OR} = -\frac{1}{2}(2\underset{\sim}{i} + 2\underset{\sim}{j} - 4\underset{\sim}{k}) + \frac{3}{2}(6\underset{\sim}{i} + 10\underset{\sim}{j} + 2\underset{\sim}{k}) \quad \text{— M1}$$

$$= (-\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}) + (9\underset{\sim}{i} + 15\underset{\sim}{j} + 3\underset{\sim}{k}) \quad \text{— M1}$$

$$= 8\underset{\sim}{i} + 14\underset{\sim}{j} + 5\underset{\sim}{k} \quad \text{— A1 (CAO).}$$

03.

Note: Allow column form for working.

9. Calculate the quartile deviation in 18, 9, 14, 20, 23, 12, 16 (3 marks)

9, 12, 14, | 16 |, 18, 20, 23

Q_1 (12) $\quad$ Q_3 (20)

$Q_1 = 12$, $Q_3 = 20$ } — B_1 for both Q_1 and Q_3

$$Q_d = \frac{20 - 12}{2} \quad \text{— M1}$$

$$= 4. \quad \text{— A1}$$

03.

(3 marks)

10. Solve for the exact value of x in the equation.

$$2\log_{10}x + \log_{10}5 = 1 + 2\log_{10}4$$

$\log_{10}x^2 + \log_{10}5 = \log_{10}10 + \log_{10}16$ — M_1

$\log_{10}5x^2 = \log_{10}160$

$5x^2 = 160$ — M_1

$\sqrt{x^2} = \sqrt{32}$

$x = 4\sqrt{2}$ — A_1 (CAO)

$\overline{03}$.

(3 marks)

11. Evaluate $\int_{-1}^{3}(2x^2 - 3x - 14)dx$

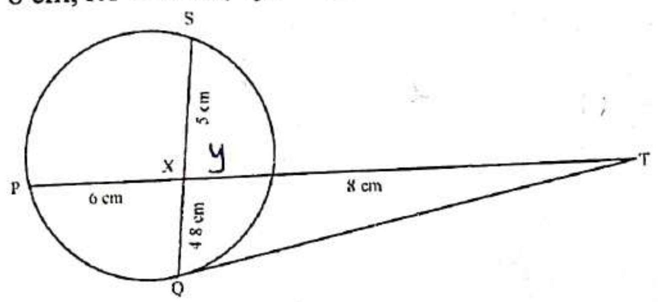
$= \left[\frac{2}{3}x^3 - \frac{3}{2}x^2 - 14x\right]_{-1}^{3}$ — M1

$= \left(\frac{2}{3}(3)^3 - \frac{3}{2}(3)^2 - 14(3)\right) - \left(\frac{2}{3}(-1)^3 - \frac{3}{2}(-1)^2 - 14(-1)\right)$ — M1

$= -49\frac{1}{3}$. — A1

$\overline{03}$.

12. In the figure below QT is a tangent to the circle at Q. PXRT and QXS are straight lines. PX = 6 cm, RT = 8 cm, QX = 4.8 cm and XS = 5 cm.



(3 marks)

Find the length QT.

$y = \frac{4.8 \times 5}{6} = 4$ cm. — B_1

$\sqrt{(QT)^2} = \sqrt{18 \times 8}$ — M_1

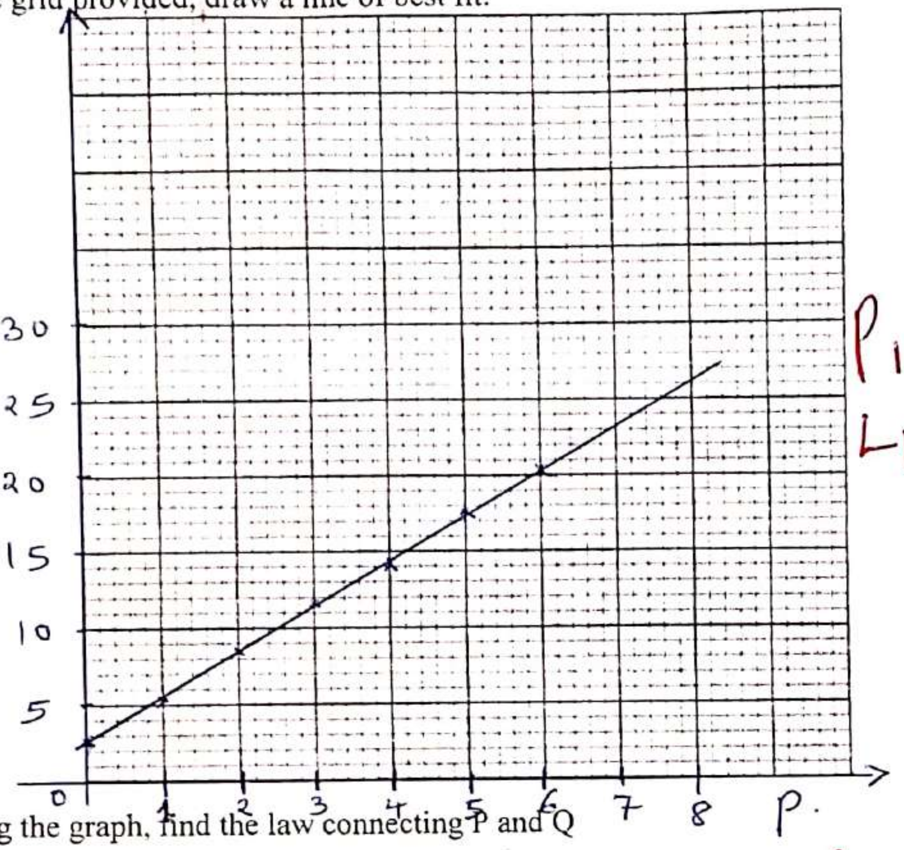
$QT = 12$ cm. — A1

$\overline{03}$.

13. The table below shows the relationship between the quantities P and Q.

P	0	1	2	3	4	5	6
Q	2.4	5.2	8.4	11.6	14.0	17.6	20.4

(a) On the grid provided, draw a line of best fit. (2 marks)



P1

L1

(b) Using the graph, find the law connecting P and Q (2 marks)

$m = \frac{20.4 - 2.4}{6 - 0} = 3.0 \pm 0.2$ — B1

$Q = 3P + 2.4$ — B1

04.

14. Radido bought a '*tuk – tuk*' whose cash price is kshs 500,000. He made a down payment of kshs. 140,000 and paid monthly installments of kshs. 25, 000 for two years. Calculate the monthly rate at which compound interest was charged. (3 marks)

$P = 500,000 - 140,000$
$= sh. 360,000$

$A = 24 \times 25,000$
$= sh. 600,000$

M1

$600,000 = 360,000\left(1 + \frac{r}{100}\right)^{24}$

$\sqrt[24]{1\tfrac{2}{3}} = \sqrt[24]{\left(1 + \frac{r}{100}\right)^{24}}$ — M1

$r = 2.151\%$. — A1

03.

15. A transformation is represented by the matrix $\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$. This transformation maps a triangle ABC of the area $12.5cm^2$ onto another triangle $A^1B^1C^1$. Find the area of triangle $A^1B^1C^1$ (3 marks)

$\det = (2 \times 1) - (4 \times 3)$ — M1

$= -10$

Area of $A'B'C' = (10 \times 12.5) cm^2$ — M1

$= 125 cm^2$ — A1

03

16. Without using tables or calculators, express $\frac{\sqrt{3}+sin30^0}{2-tan60^0}$ in the form $a + b\sqrt{c}$

(3 marks)

$\frac{(\sqrt{3} + \frac{1}{2})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$ — M1

$= \frac{2\sqrt{3} + 3 + 1 + \frac{1}{2}\sqrt{3}}{4 - 3.}$ — M1

$= 4 + 2\frac{1}{2}\sqrt{3}$ — A1

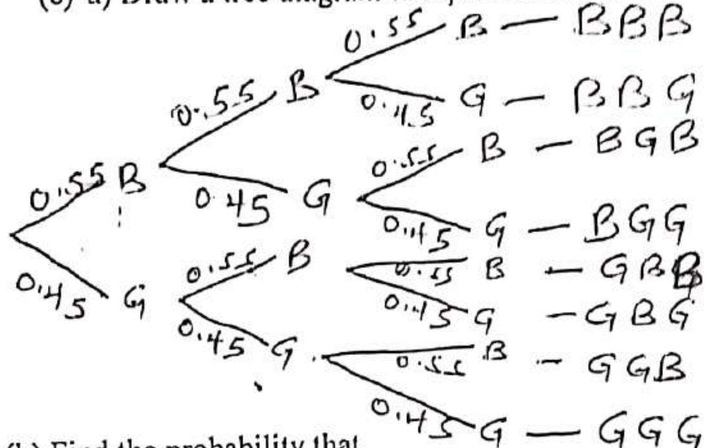
03.

SECTION II(50marks)

Answer any five questions

17. A married couple intends to have 3 children. They consult an expert who tells them that the probability of a male birth is 0.55

(c) a) Draw a tree diagram to represent this occurrence. (2mrks)



2

(b) Find the probability that (2marks)

(i) all the three children will be female.

P(GGG)

$0.45 \times 0.45 \times 0.45 = \underline{0.091125}$ M1 A1

02

(ii) At least a male is born. (2marks)

$1 - P(GGG)$

$1 - 0.091125$ M1

0.908875 A1

02

(iii) At least 2 will be females, giving your answer to 3 s.f. (4marks)

P(BGG) or P(GBG) or P(GGB) or P(GGG)

$(0.55 \times 0.45 \times 0.45) + (0.45 \times 0.55 \times 0.45) + (0.45 \times 0.45 \times 0.55) + (0.45 \times 0.45 \times 0.45)$

$0.11135 + 0.111375 + 0.111375 + 0.091125$ M1

$0.334125 + 0.091125$

0.42525

$= \underline{0.425}$ A1

04

10

18. A trader is required to supply two types of sweaters, type **A** and type **B**. The total number of sweaters must be more than 400. He has to supply more of type A than type **B** sweaters. However the number of type **A** sweaters must not be more than 300 and the number of type **B** sweaters must not be less than 80. Let **x** be the number of type **A** sweaters and y be the number of type **B** sweater.

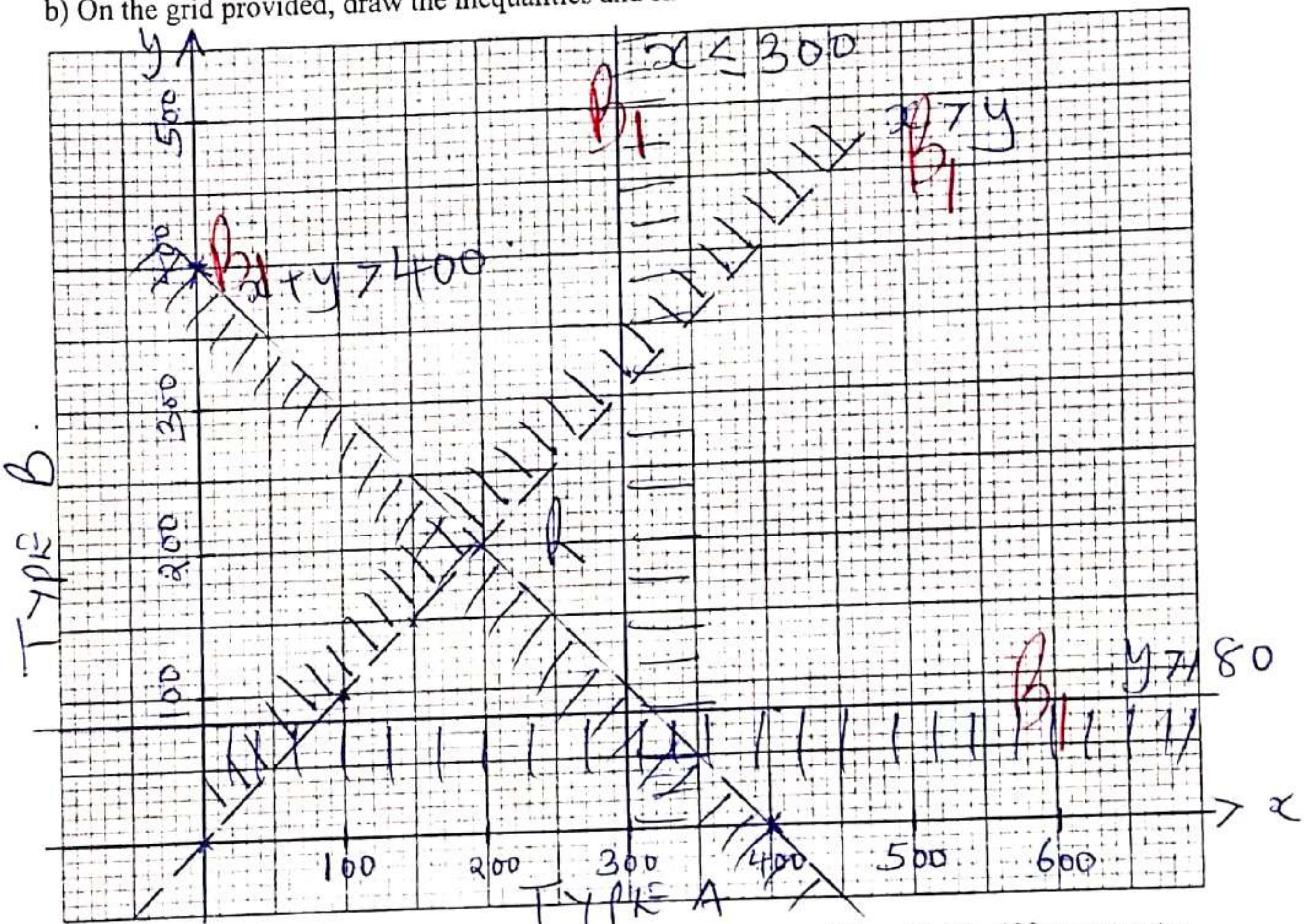
(a) Write down in terms of **x** and **y** all the linear inequalities representing the information above. (4 marks)

$x + y > 400$ — B1 $\quad y \geq 80$ · — B1

$x > y$ ——— B1

$x \leq 300$ ——— B1

b) On the grid provided, draw the inequalities and shade the unwanted regions. (4 marks)



c) The profits were as follows; Types A: Sh.600 per sweater and Type B: Sh. 400 per sweater. Use the graph to determine the number of sweaters of each type that he should make to maximize the profit. Hence calculate the maximum possible profit. (2 marks)

Type A = 300, Type B = 290 } B1

$P = 300(600) + 290(400)$

$= $ Sh. 296,000 · — B1

9

10

19.The table shows income tax rates for a certain year.

Monthly taxable pay K£	Rate of tax in Ksh. Per K£
1 – 434	2
435 – 866	3
867 – 1298	4
1299 – 1730	5
OVER 1730	6

A company employee earns a monthly basic salary and is given taxable allowances amounting to Ksh 10480. If the employees' tax on the 5^{th} band is ksh 3420,

(a) Calculate the employee's monthly taxable income tax in K£. (2mrks)

$\frac{(x-1730)6}{6} = \frac{3420}{6}$ M1

$x = 570+1730$

Ksh 2300 A1

02

(b) The employee is entitled to personal tax relief of ksh. 1056 per month. Determine the net tax (4marks)

$1^{st} \Rightarrow 434\times2 \rightarrow 868$

$2^{nd} \Rightarrow 432\times3 \rightarrow 1296$ B1

$3^{rd} \Rightarrow 432\times4 — 1728$ B1

$4^{th} \Rightarrow 432\times5 — 2160$

5^{th} 3420 $= 9472$

$9472-1056$ M1

$= 8416$ A1

04

(c) In a certain month, the employee received a 25% increment in his basic salary. Calculate his net monthly pay (4mrks)

TI = BS + ALL

$(2300\times20) = BS + 10480$

$46000 - 10480 = BS$

$35520 \times \frac{125}{100}$ M1

$= 44,400$

$TI = 44400+10480$

$= 54880$

NS = TI − P

$54880-11046$

$= \cancel{43834}$

M1

$5^{th} \Rightarrow 6\times1014 \Rightarrow 6084$

PAYE $= 12102-1086$

$= 11046$ A1

$54880-11046$

43834 B1

04

10

12

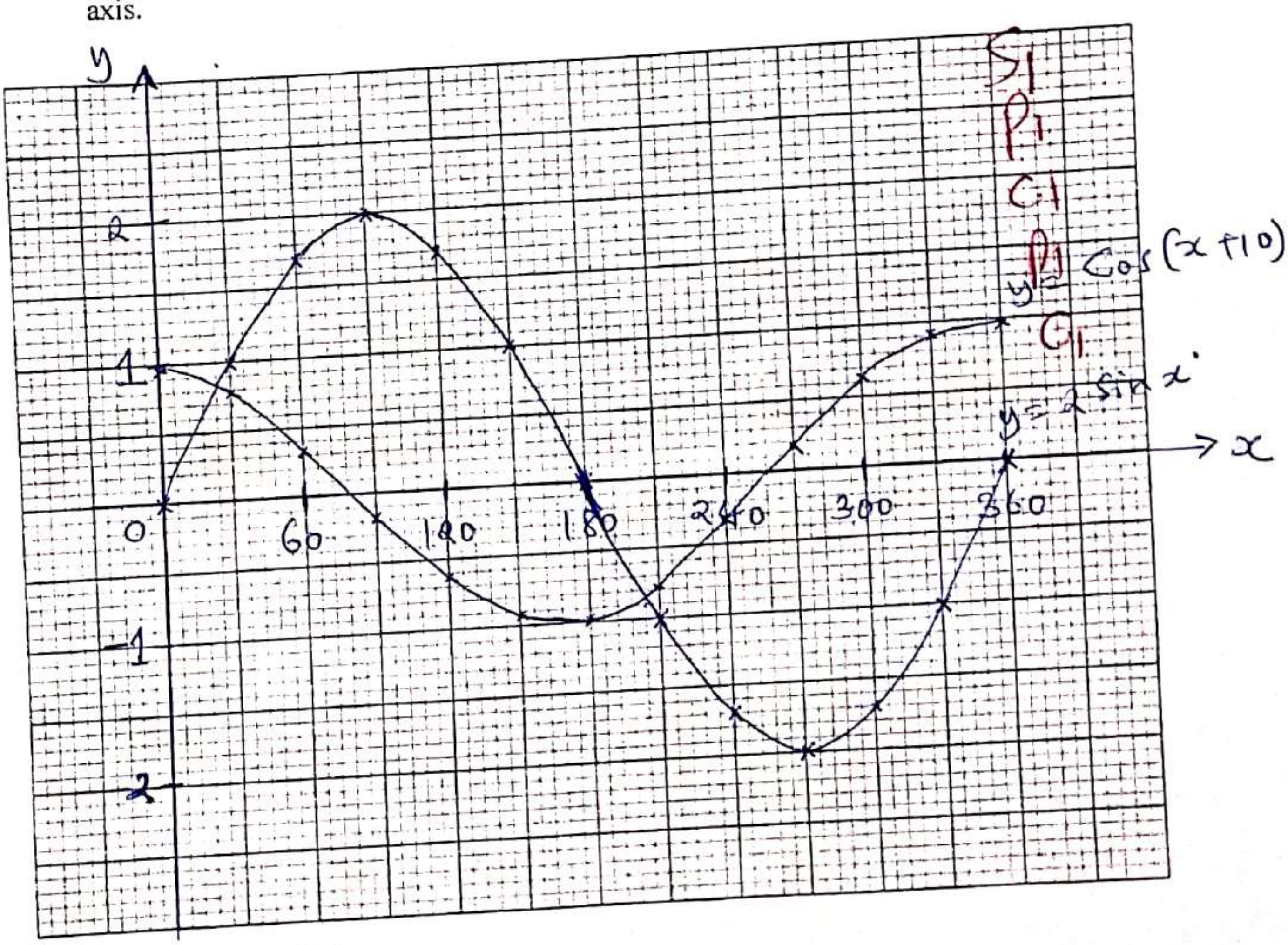
20. Given that y = 2 sinx^0 and y = cos $(x + 10)^0$

(a) Complete the table below correct to 2decimal places (2marks)

X	0	30	60	90	120	150	180	210	240	270	300	330	360
$2sinx^0$	0	1.00	1.73	2.00	1.73	1	0	-1.00	-1.73	-2.00	-1.73	-1.00	0
$\cos(x + 10)^0$	0.98	0.77	0.34	-0.17	-0.64	-0.94	-0.98	-0.77	-0.34	0.17	0.64	0.94	0.98

B_2 - All values
B_1 - Atleast half.

(b) On the same axes, draw the graphs y = $2sinx^0$ and y =$\cos(x + 10)^0$ for $0^0 \leq x \leq 360^0$. Using the scale: 1cm rep 30^0on the horizontal axis and 2cm rep 1cm unit on the vertical axis. (5 marks)



(c)Use your graph to find:

(i) The range of values of x for which $2sinx^0 \geq cos(x + 10)^0$ (2 marks)

$24^\circ \leq x \leq 204^\circ \pm 3^\circ$ B_1 B_1

(ii) The period of y = $2sinx^0$ (1 mark)

360° B_1

10

21 The positions of two towns on the surface of the earth are given as A $(45^0 s, 20^0 w)$ and B $(45^0 S, 80^0 E)$.

Find:

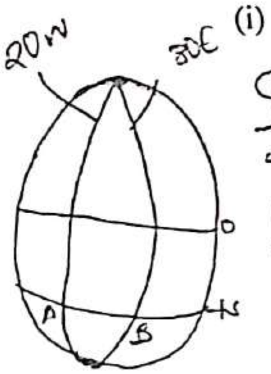
a) The difference in longitude (1mrks)

$80 + 20 = 100°$ B1

0 1

b) The distance between the two towns in : (3mrks)

(i) Kilometers (R =6370 AND $\pi = \frac{22}{7}$)



$\frac{\theta}{360} 2\pi R \cos\alpha$

$\frac{100}{360} \times 2 \times \frac{22}{7} \times 6370 \cos 45$ M1

$\underline{\underline{7864.60 \text{ km}}}$ A1

0 3

(ii) Nautical miles (3mrks)

$\theta \times 60 \cos\alpha$

$\Rightarrow 100 \times 60 \cos 45$ M1

$\Rightarrow \underline{\underline{4242.64 \text{ nm}}}$ A1

2

c) The local time in town A when it is 8.20am in B (3mrks)

$1° = 4 \text{min}$

$100 = ?$

$\frac{100 \times 4}{6} = \frac{400}{60}$ M1

$\underline{\underline{6h\ 40m}}$

1200
820
2020
640 M1
1340 hrs

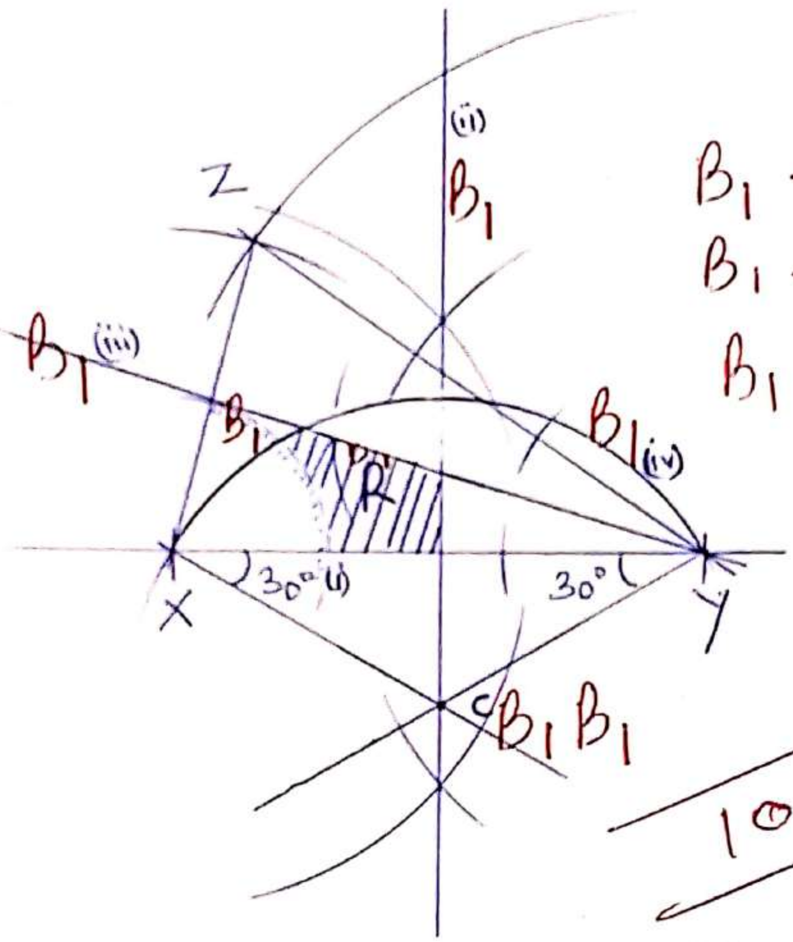
$\underline{\underline{= 1{:}40 \text{pm}}}$ A1

0 3

10

22. Construct a triangle XYZ in which XY=7cm, YZ=7.2cm and XZ=4.2cm. Measure angle XYZ

(3 marks)



$\angle XYZ = 34° \pm 1°$ B_1

B_1 – X, Y and Z Located

B_1 – Complete diagram

B_1 – R shaded.

10

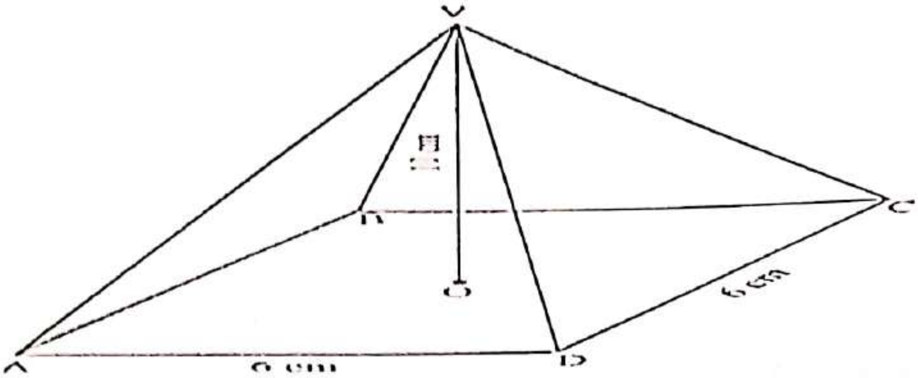
A point R is located inside triangle XYZ such that:

(i) XR> $2cm$(1mrk)
(ii) R is closer to X than Y
(iii) R is closer to XY than YZ
(iv) $< XRY \geq 120^0$
(v) Shade hc locus of R

As on the diagram

(7 marks)

23. The figure below shows a square based pyramid ABCDV with AD = DC =6cm and OV = 10CM



(a) State the projection of VA on base ABCD. (1mrks)

AO

(b)Find;

(i)The length VA. (3mrks)

$\frac{1}{2}AC = \frac{\sqrt{6^2+6^2}}{2} = \frac{8.485}{2} = 4.243$

$AV = \sqrt{10^2+4.243^2}$

10.86

(ii)The angle between VA and ABCD. (2mrks)

$\sin\theta = \frac{10}{10.86} = 74.49°$

(iii)the angle between VDC and ABCD (2mrks)

$\tan\theta = \frac{10}{3} = 73.30$

(iv)the volume of the pyramid (2mrks)

$(\frac{1}{3} \times 6 \times 6 \times 10)$

$= 120cm^3$

10

24. In an arithmetic progression (A.P), the 4^{th} term and 9^{th} term are 30 and 45 respectively.

(a) Find the first term and common difference of the A.P. (3mrks)

$a+3d = 30$
$- \quad a+8d = 45$
$-5d = -15$
$d = 3$

$a+3d = 30$
$a = 30-9$
$a = 21$ A1

03

(b) The second, sixth and twelfth terms of the above A.P are the first three consecutive terms of a geometric progression (G.P). Find:

(i) The common ratio. (3mrks)

$2^{nd} \rightarrow 21+3 = 24$
$6^{th} \rightarrow 21+15 = 36$
$12^{th} \rightarrow 21+33 = 54$ B1

$r = \frac{36}{24} = \frac{54}{36}$ M1

$r = 1.5$ A1

03

(ii) The fourth term of the G.P. (2mrks)

$n^{th} = ar^{n-1}$
$= 24(1.5)^3$ M1
$= 81$ A1

02

(iii)The sum of the first 5terms of the G.P. (2mrks)

$Sum = \frac{a(r^n-1)}{r-1}$

$= \frac{24(1.5^5-1)}{0.5}$ M1

$= 316.5$ A1

02