NATIONAL MATHS LINK JOINT EXAM - 2024

Kenya Certificate of Secondary Education

MATHS LINK JOINT EXAM

121/2 **MATHEMATICS Alt. A** **Paper 2**

Jan. 2024 – 2½ Hours

Exam Date; 2 / 2 / 2024

Name: ....MARKING GUIDE......................... Adm Number: ....

Student's Signature: .... School: .... Class:.........

Instructions to candidates

(a) Write your name, Adm number and class in the spaces provided above.
(b) Sign and write the date of examination in the spaces provided above.
(c) This paper consists of **two** sections: **Section I** and **Section II**.
(d) Answer all the questions in Section **I** and only **five** questions from **Section II**.
(e) **Show all the steps in your calculations, giving your answers at each stage in the spaces provided below each question.**
(f) Marks may be given for correct working even if the answer is wrong.
(g) **Non – programmable** silent electronic calculators and KNEC Mathematical tables may be used, except where stated otherwise.
(h) **This paper consists of 15 printed pages.**
(i) **Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.**
(j) **Candidates should answer the questions in English.**

For Examiner's Use Only

Section I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

Section II

17	18	19	20	21	22	23	24	Total

Grand Total

SECTION I (50 Marks)

*Answer **all** the questions in this section in the spaces provided.*

1. Mark took the measurements of his room and gave the length as 9 m and width as 7 m. If there is an error of 2% in each measurement, determine the range within which the perimeter of the room lies. *Give your answer correct to 4 decimal places.* (3 marks)

Maximum Perimeter $= 2(9.18 + 7.14)$ ✓ M1

$= 32.64$ cm.

Minimum Perimeter $= 2(8.82 + 6.86)$ ✓ M1

$= 31.36$ cm.

$31.36\text{cm} \leq$ Perimeter ≤ 32.64 cm. ✓ A1

2. A transformation matrix $N = \begin{pmatrix} x & 3 \\ -x & x-2 \end{pmatrix}$ maps a triangle whose area is 2.5 cm^2 onto a triangle whose area is 15 cm^2. Find the possible values of x. (3 marks)

$x(x-2) + 3x = \frac{15}{2.5}$. ✓ M1

$x^2 - 2x + 3x = 6.$

$x^2 + x - 6 = 0$

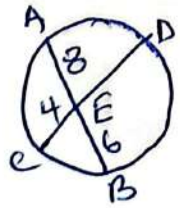
$x^2 - 2x + 3x - 6 = 0$

$x(x-2) + 3(x-2) = 0$ ✓ M1

$(x-2)(x+3) = 0$

$x = 2$ or -3. ✓ A1

3. Chords AB and CD of a circle intersect at a point E inside the circle. Given that EA = 8 cm, AB = 14 cm and EC = 4 cm, calculate the length of ED. (2 marks)



$AE \times EB = CE \times ED$

$8 \times 6 = 4 \times ED$. ✓ M1

$\frac{48}{4} = ED$

$ED = 12$cm. ✓ A1

4. Given that $A = \sqrt[3]{\frac{d-c^2}{b+c^2}}$, make c the subject of the formula. (3 marks)

$$\frac{A^3}{1} = \frac{d-c^2}{b+c^2}$$ ✓ M1

$$d - c^2 = A^3 b + A^3 c^2$$

$$A^3c^2 + c^2 = d - A^3 b.$$

$$c^2 = \frac{d - A^3 b}{A^3 + 1}$$ ✓ M1

$$c = \pm\sqrt{\frac{d - A^3 b}{A^3+1}}$$ ✓ A1

5. Solve for x in the equation; $(\log_{10} x)^2 + \log x^2 - 3 = 0$ (3 marks)

Let $\log_{10} x$ be p.

$$p^2 + 2p - 3 = 0.$$ ✓ M1

$$p = -3 \text{ or } 1$$

$$\log_{10} x = -3 \text{ or } 1$$ ✓ M1

$$x = 10^{-3} \text{ or } 10^1$$

$$x = \frac{1}{1000} \text{ or } 10$$ ✓ A1

6. Solve the simultaneous equations; $\begin{matrix} 4x - y = 7 \\ xy = 15 \end{matrix}$. (4 marks)

From eqn (i) $y = 4x - 7$ ✓ Subst. M1

$$x(4x-7) = 15$$

$$4x^2 - 7x - 15 = 0$$ ✓ Solving process M1

$$x = -1.25, 3$$ ✓ First unknown A1

When $x = -1.25$, $y = -12$

When $x = 3$, $y = 5$. Both values. B1

7. (a) Expand $(2x-3y)^4$ (1 mark)

$$1.(2x)^4(3y)^0 - 4(2x)^3(3y)^1 + 6(2x)^2(3y)^2 - 4(2x)^1(3y)^3 + 1.(2x)^0(3y)^4$$

$$16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4 \checkmark$$ B1

(b) Hence use a suitable substitution to estimate $(2.03)^4$ correct to 4 decimal places. (2 marks)

$2x = 2 \Rightarrow x = 1$ and $-3y = -0.03$, $y = 0.01$

$$16(1)^4 - 96(1)^3(0.01) + 216(1)^2(0.01)^2 - 216(1)(0.01)^3 + 81(0.01)^4 \checkmark$$ M1

$$16 - 0.96 + 0.0216 - 0.000216 + 0.00000081$$

$$= \underline{15.0614} \checkmark$$ A1

8. Solve the equation $\sin\left(4\frac{1}{2}x - \frac{1}{4}\pi^c\right) = \frac{\sqrt{2}}{2}$ for $0 \le x \le \pi^c$ leaving your answers in terms of π^c (3 marks)

$$\left(4.5x - \tfrac{1}{4}\pi^c\right) = 45^\circ \checkmark$$

$$= 45^\circ, 135^\circ, 405^\circ, 495^\circ, 765^\circ, 855 \checkmark$$ M1 M1

$$4.5x = 90, 180, 450, 540, 810, 900$$

$$x = \tfrac{1}{9}\pi^c, \tfrac{2}{9}\pi^c, \tfrac{5}{9}\pi^c, \tfrac{2}{3}\pi^c, \pi^c \checkmark$$ A1

9. Calculate the variance of the following list of values giving your answer correct to four significant figures. 4, 5, 7, 8, 9, 15 (3 marks)

Mean; $\frac{4+5+7+8+9+15}{6} = 8$

$d^2 = (x-\bar{x})^2 = 16, 9, 1, 0, 1, 49 \checkmark$ M1

$s^2 = \frac{\Sigma(x-\bar{x})^2}{N} = \frac{76}{6} \checkmark$ M1

$= 12\frac{2}{3} \checkmark$ Accept 12.67 A1

10. A quantity P varies partly as the cube of Q and partly varies inversely as the square of Q. When Q = 2, P = 108 and when Q = 3, P = 259. Find the value of P when Q = 4. (4 marks)

$P \propto Q^3 + \frac{1}{Q^2}$

$P = aQ^3 + b/Q^2$

$8a + \frac{b}{4} = 108$ ✓ $\rightarrow$ $32a + b = 432$

$27a + \frac{b}{3} = 259$ $\quad$ $81a + b = 777$ −

$-49a = -345$

$a = 7\frac{2}{49}$ ✓

$b = 206\frac{34}{49}$

M1

M1

A

$a = 7\frac{2}{49}$ or 7.041 ✓ $b = 206\frac{34}{49}$ or 206.69; $P = 502.30$ ✓ B1

11. The sum of the first 4 terms of an arithmetic progression is 158 and the sum of the first 7 terms is 154. Find the first term and the common difference. (3 marks)

$S_n = \frac{n}{2}(2a + (n-1)d)$.

$2(2a + 3d) = 158$

$3.5(2a + 6d) = 154$ ✓

$2a + 3d = 79$ −

$2a + 6d = 44$ ✓

$-3d = 35$

$d = -\frac{35}{3}$

$a = 57$

$a = 57$; $d = -11\frac{2}{3}$ ✓

M1

M1

A1

12. Twenty years ago, the area under forest cover in a certain county was 288 000 ha. Due to deforestation, the area under the forest cover has reduced to 240 000 ha. Calculate the rate of deforestation in the county per decade. (3 marks)

(3 marks)

$A = P\left(1 - \frac{r}{100}\right)^n$.

$288\,000\left(1 - \frac{r}{100}\right)^2 = 240\,000$ ✓

$1 - \frac{r}{100} = 0.91287093$

$100 - r = 91.287093$ ✓

$r = 8.71207\%$

$r = 8.712\%$ ✓ At least 4 s.f

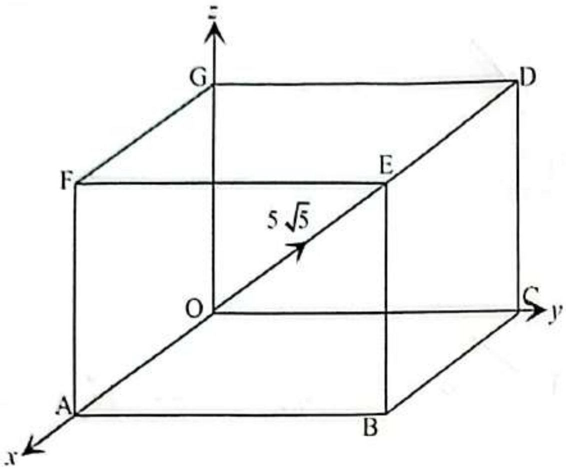
M1

M1

A1

13. In the figure below, $A = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix}$ and $C = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix}$. The distance between E and the origin is $5\sqrt{5}$ units.

Determine the coordinates of E.



$$\underset{\sim}{OE} = \begin{pmatrix} 5 \\ 6 \\ z \end{pmatrix}.$$

$$|\underset{\sim}{OE}| = \sqrt{25 + 36 + z^2}$$

$$\therefore 25 + 36 + z^2 = 125$$ ✓ M1

$$z^2 = 64$$

$$z = \pm 8$$

$$z = 8$$ ✓ A1

$E(5, 6, 8)$. ✓ B1

14. (a) Expand and simplify $\left(\sqrt{2}+2\sqrt{3}\right)\left(4\sqrt{2}-\sqrt{3}\right)$, leaving your answer in the form $a+b\sqrt{c}$ where a, b and c are integers. (2 marks)

$$\sqrt{2}\left(4\sqrt{2}-\sqrt{3}\right) + 2\sqrt{3}\left(4\sqrt{2}-\sqrt{3}\right)$$ ✓ M1

$$8 - \sqrt{6} + 8\sqrt{6} - 6.$$

$$2 + 7\sqrt{6}.$$ ✓ A1

(b) Hence rationalize the denominator of the fraction below, leaving your answer in its simplified form. (2 marks)

$$\frac{-580}{\left(\sqrt{2}+2\sqrt{3}\right)\left(4\sqrt{2}-\sqrt{3}\right)}$$

$$\frac{-580\left(2-7\sqrt{6}\right)}{\left(2+7\sqrt{6}\right)\left(2-7\sqrt{6}\right)}$$ ✓ M1

$$\frac{-1160 + 4060\sqrt{6}}{4 - 294}$$

$$4 - 14\sqrt{6}.$$ ✓ A1

15. The equation of a circle is $x^2+y^2+kx+6y-12=0$ where k is an integer. Given that the radius of the circle is 5 units, find the possible values of k. (3 marks)

$$x^2+kx+c_1+y^2+6y+c_2=12$$

$$x^2+kx+\left(\frac{k}{2}\right)^2+y^2+6y+(3)^2=12+\left(\frac{k}{2}\right)^2+9$$

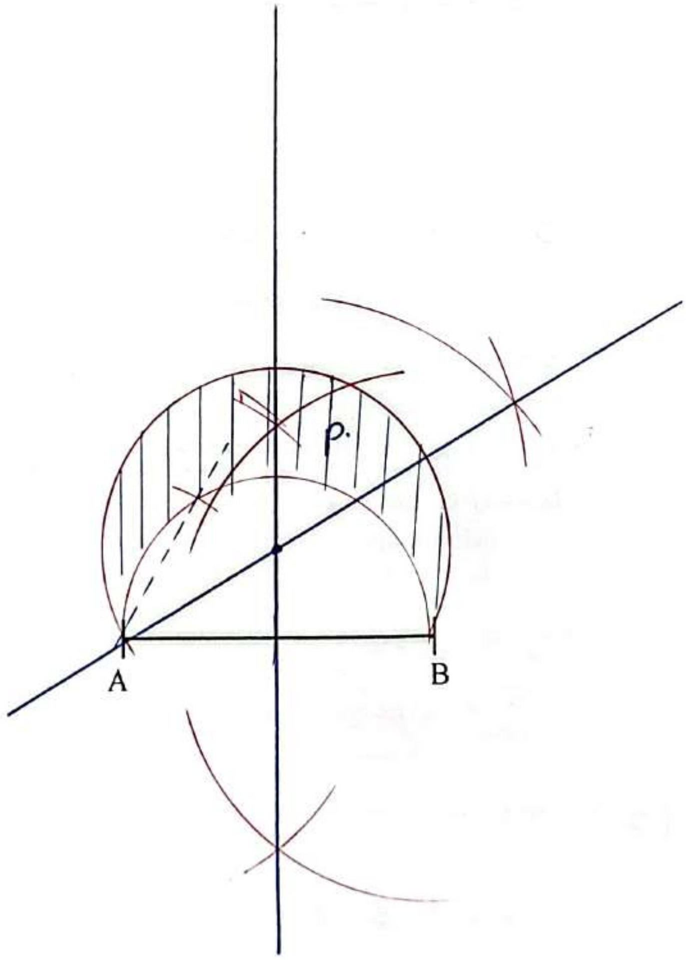
$$\left(x+\frac{k}{2}\right)^2+(y+3)^2=21+\frac{k^2}{4} \quad \checkmark \qquad \text{M1}$$

$$21+\frac{k^2}{4}=25 \quad \checkmark \qquad \text{M1}$$

$$k^2=16$$

$$k=\pm 4 \quad \checkmark \qquad \text{A1}$$

16. Using the upper side of the line AB below, shade and label the region P enclosed by the locus of P such that $60^\circ < \angle APB \leq 90^\circ$ (4 marks)



90° B1

centre B1

60° B1

Locating P B1

SECTION II (50 Marks)

Answer ALL questions from this section in the spaces provided.

17. The table below shows the income tax rates for a certain year.

Monthly table income in Kenya shillings	Tax rate in each shilling
10101 – 19800	10%
19801 – 28800	15%
28801 – 39200	20%
39201 – 48700	25%
Over 48700	30%

A tax relief of Kshs.1162 per month was allowed. In a certain month, of that year, an employee's taxable income in the fourth band was Kshs.3 800.

(a) Calculate;

(i) The employee's total taxable income in that month; (2 marks)

$39\,200 + 3\,800$ M1

$= \text{Kshs. } 43{,}000$ A1

(ii) The tax payable by the employee in that month. (5 marks)

$9700 \times \frac{10}{100} = \text{Kshs. } 970$

$9000 \times \frac{15}{100} = \text{Kshs. } 1350$ M1

$10400 \times \frac{20}{100} = \text{Kshs. } 2080$ M1

$3800 \times \frac{25}{100} = \text{Kshs. } 950$

Total Tax $= \text{Kshs. } 5350$ A1

Taxable Income $= 5350 - 1162 = \text{Kshs } 4188$ M1 A1

(b) The employee's income included a house allowance of Kshs.12 000 per month. The employee contributed 5% of the basic salary to WCPS. Calculate the employee's net pay for that month. (3 marks)

Basic salary $= 43000 - 12000 = 31\,000$.

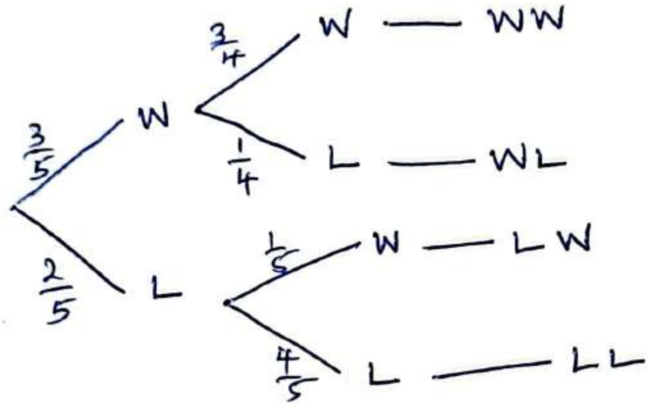
WCPS $= \frac{5}{100} \times 31000 = \text{Kshs. } 1{,}550$. M1

Net pay $= 43\,000 - (4188 + 1550)$ M1

$= \text{Kshs. } 37\,262$ A1

18. In a volleyball competition, each team plays two matches. The probability that a team wins the first match is $\frac{3}{5}$. If it wins the first match, the probability that it wins the second match is $\frac{3}{4}$ otherwise it is $\frac{1}{5}$

(a) Represent the information on a tree diagram. (2 marks)



B2

(b) Find the probability that;

(i) A team wins both matches (2 marks)

P(WW)

$\frac{3}{5} \times \frac{3}{4} = \frac{9}{20}$

M1
A1

(ii) A team wins a single match. (3 marks)

P(WL) or P(LW)

$\left(\frac{3}{5} \times \frac{1}{4}\right) + \left(\frac{2}{5} \times \frac{1}{5}\right) = \frac{23}{100}$

M1
M1
A1

(iii) A team does not win. (2 marks)

P(LL)

$\frac{2}{5} \times \frac{4}{5} = \frac{8}{25}$

M1
A1

(iv) A team wins the second match only. (2 marks)

P(LW)

$\frac{2}{5} \times \frac{1}{5}$

$= \frac{2}{25}$

M1
A1

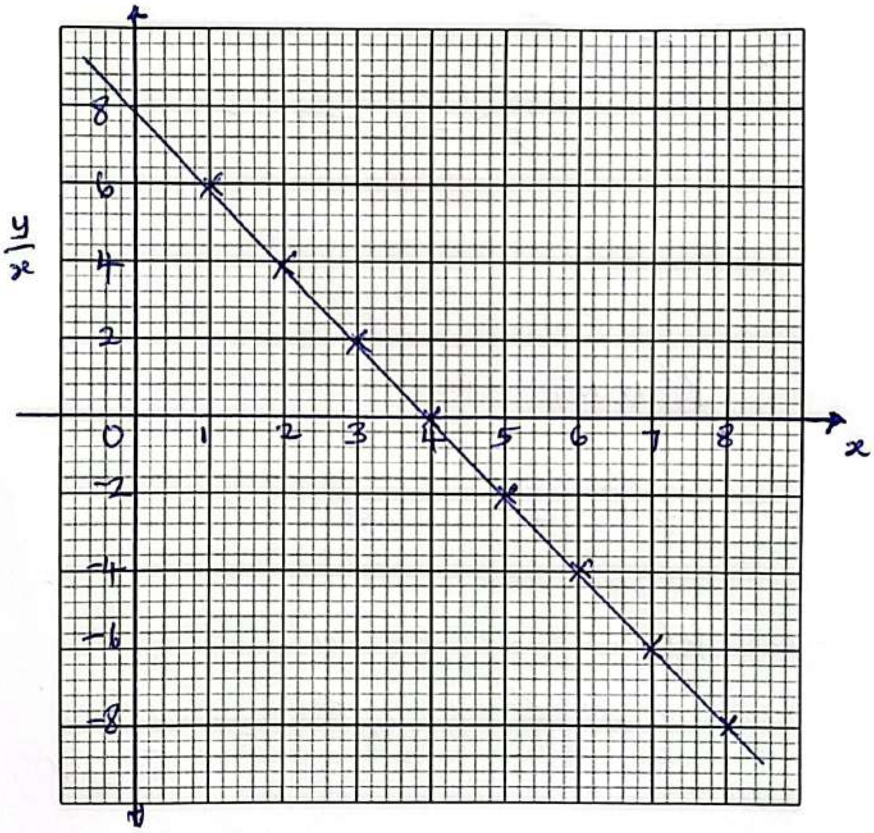
19. Two variables x and y are believed to obey the law $y = mx + nx^2$ where m and n are constants. The following table gives their corresponding values in an experiment.

x	1	2	3	4	5	6	7	8
y	6	8	6	0	−10	−24	−42	−64
$\frac{y}{x}$	6	4	2	0	−2	−4	−6	−8

B2

(a) Complete the table above for the values of $\frac{y}{x}$. (2 marks)

(b) Draw the graph of $\frac{y}{x}$ (y – axis) against x (x – axis) on the grid provided below. Use a scale of 1 cm to represent 1 unit on the x – axis and 1 cm to represent 2 units on the y – axis. (3 marks)



S1
P1
C1

(c) Determine the values of the constants m and n. (3 marks)

$\frac{y}{x} = nx + m$.

$m = 8$

$n = \frac{2-4}{3-2} = -\frac{2}{1} = -2$

B1
M1
A1

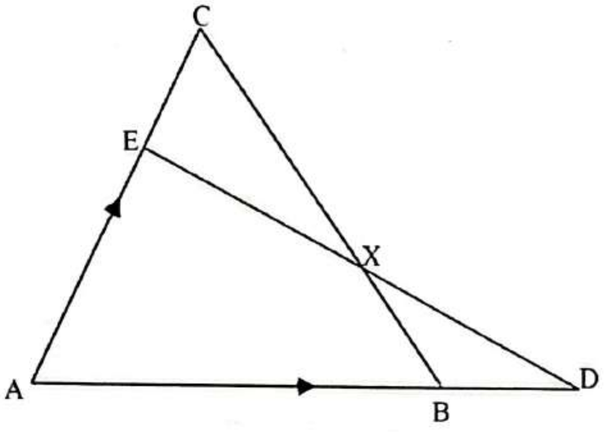
(d) Use the law connecting x and y to determine the value of y when x is 20. (2 marks)

$\frac{y}{x} = -2x + 8$ $\quad$ $\frac{y}{20} = -40 + 8$

$y = -32 \times 20 = -640$.

B1
B1

20. In the figure below **AB** = **a**, **AC** = **b**, **AE : EC** = 4:1 and **AB : BD** = 5:3. ED and CB intersect at X such that CB = hCX and EX = kED where h and k are scalars.



(a) Express the following vectors in terms of a and b: (2 marks)

(i) CB

$\underset{\sim}{CB} = \underset{\sim}{CA} + \underset{\sim}{AB}$

$= -\underset{\sim}{b} + \underset{\sim}{a}$. B1

(ii) ED

$\underset{\sim}{ED} = \underset{\sim}{EA} + \underset{\sim}{AD}$.

$= -\frac{4}{5}\underset{\sim}{b} + \frac{8}{5}\underset{\sim}{a}$ B1

(b) (i) Express AX in two different ways. (2 marks)

$\underset{\sim}{AX} = \underset{\sim}{AC} + \underset{\sim}{CX}$.

$= \underset{\sim}{b} + \frac{1}{h}(-\underset{\sim}{b} + \underset{\sim}{a})$

$= \left(1 - \frac{1}{h}\right)\underset{\sim}{b} + \frac{1}{h}\underset{\sim}{a}$ B1

$\underset{\sim}{AX} = \underset{\sim}{AE} + \underset{\sim}{EX}$,

$= \frac{4}{5}\underset{\sim}{b} + k\left(-\frac{4}{5}\underset{\sim}{b} + \frac{8}{5}\underset{\sim}{a}\right)$

$= \left(\frac{4}{5} - \frac{4}{5}k\right)\underset{\sim}{b} + \frac{8}{5}k\underset{\sim}{a}$. B1

(ii) Determine the values of h and k. (4 marks)

$1 - \frac{1}{h} = \frac{4}{5} - \frac{4}{5}k$

$\frac{1}{h} = \frac{8}{5}k$. M1

$h = \frac{5}{8k}$

$1 - \frac{8k}{5} = \frac{4}{5} - \frac{4}{5}k$ M1

$5 - 8k = 4 - 4k$

$4k - 8k = 4 - 5$

$-4k = -1$ M1

$k = \frac{1}{4}$

$h = \frac{5}{2}$

$= 2\frac{2}{5}$ A1

(c) Show that E, X and D are collinear. (2 marks)

$\underset{\sim}{EX} = \frac{5}{2}\underset{\sim}{ED}$ B1

EX // ED with point E common, hence E, X and D are collinear. B1

21. (a) The second term of an arithmetic progression is a fifth of its thirteenth term. The sum of the first twelve terms of the same progression is 348.

(i) Find the first term and the common difference of the progression. (3 marks)

$a+d=\frac{1}{5}(a+12d)$ ✓ M1

$a=\frac{7}{4}d$ M1

$6(2a+11d)=348$ ✓

$14d+44d=232$ A1

$a=7,\ d=4$ ✓

(ii) Given that the n^{th} term of the progression is less than 57, find the greatest value of n. (3 marks)

$7+(n-1)4<57$ ✓ M1

$4(n-1)<50$ M1

$n-1<12.5$

$n<13.5$ ✓ A1

$n=13$ ✓

(b) At ICIPE Research Station, it was found that the number of weevils doubles in every one day. If the number of weevils was 100 on 15th November 2010, find the day in which the number of weevils will reach 1 000 000. (4 marks)

$S_n=\frac{a(r^n-1)}{r-1}$

$\frac{100(2^n-1)}{1}=1000\,000$ ✓ M1

$2^n=10001$

$n=\frac{\log 10001}{\log 2}$ ✓ M1

$=13.29$ ✓ A1

28th November 2010 ✓ B1

22. (a) The speed V m/s of a moving particle is partly constant and partly varies as time t seconds. It is given that $V = 28$ m/s when $t = 2$ and $V = 53$ m/s when $t = 7$ seconds. Find the speed of the particle when $t = 11$ seconds. (4 marks)

$$V = n + mt$$

$$m + 2n = 28$$
$$m + 7n = 53$$
$$5n = 25$$
$$n = 5, \quad m = 18.$$
$$V = 18 + 5t$$
$$V = 18 + (5 \times 11)$$
$$= 73 \text{ m/s}.$$

M1
A1
M1
A1

(b) A quantity R varies directly as T and inversely as the cube root of S. Given that $S = 64$ when $T = 6$ and $R = 30$;

(i) Find the formula connecting R, S and T. (3 marks)

$$R = \frac{KT}{\sqrt[3]{S}}$$
$$30 = \frac{6K}{\sqrt[3]{64}}$$
$$K = 20$$
$$R = \frac{20T}{\sqrt[3]{S}}.$$

M1
A1
B1

(ii) Find the percentage change in R when T is decreased by 10% and S increased by 25%. (3 marks)

$$R_1 = \frac{0.9 \times KT}{\sqrt[3]{1.25S}}$$

$$\% \Delta \text{ in } R = \frac{0.8357 - 1}{1} \times 100$$
$$= -16.43\%.$$

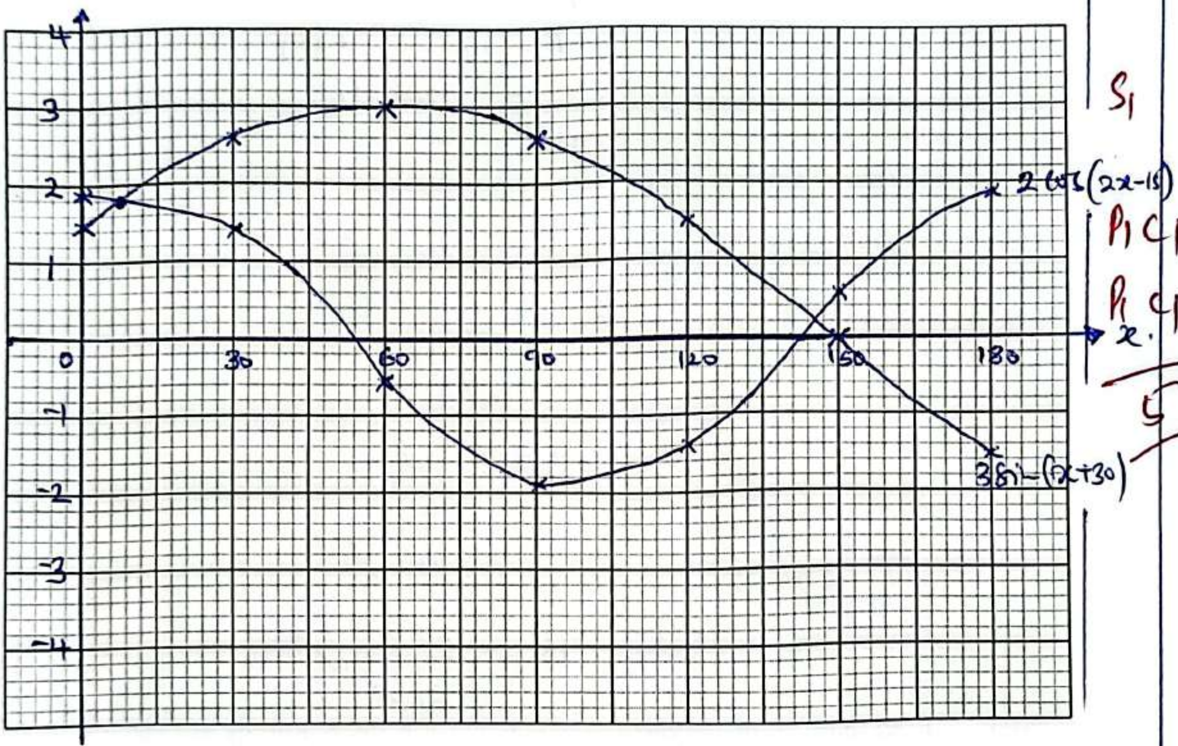
M1
M1
A1

23. (a) Complete the table below giving your values correct to two decimal places. (2 marks)

x	0	30	60	90	120	150	180
$2\cos(2x-15)$	1.93	1.41	−0.52	−1.93	−1.41	0.52	1.93
$3\sin(x+30)$	1.50	2.60	3.00	2.60	1.50	0.00	−1.5

B2

(b) Using the grid provided, draw graph of $y = 2\cos(2x-15)$ and $y = 3\sin(x+30)$ for $0^\circ \leq x \leq 180^\circ$. Use the scale of 1 cm for 1 unit on the vertical axis and 2 cm for 30 on the horizontal axis. (5 marks)



S1
P1 C1
P1 C1
5

(c) Use the graph in (b) above to solve the equation $3\sin(x+30) = 2\cos(2x-15)$ (2 marks)

$x = 7.5,\ 145.5$ ✓

B2

(d) Find the periodic time of $y = 2\cos(2x-15)$ (1 mark)

180° ✓

B1

24. Triangle $P'Q'R'$ with vertices at $P'(9,-4)$, $Q'(18,-9)$ and $R'(15,-11)$ is the image of triangle PQR under a transformation represented by the matrix $\begin{pmatrix} 3 & 0 \\ -2 & 1 \end{pmatrix}$.

(a) Find the coordinates of triangle PQR. (3 marks)

Inverse $= \frac{1}{3}\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$ M1

$\frac{1}{3}\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 9 & 18 & 15 \\ -4 & -9 & -11 \end{pmatrix} = \begin{pmatrix} 3 & 6 & 5 \\ 2 & 3 & -1 \end{pmatrix}$ ✓ ✓ M1

$P(3,2)$ $Q(6,3)$ $R(5,-1)$ ✓ A1

(b) Triangle $P''Q''R''$ is the image of triangle $P'Q'R'$ under a transformation represented by matrix **N**. Given that $P''(1,-4)$, $Q''(0,-9)$ and $R''(-7,-11)$

(i) Determine the matrix **N**. (3 marks)

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 9 & 18 \\ -4 & -9 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -4 & -9 \end{pmatrix}$ ✓

$\begin{pmatrix} 9a-4b & 18a-9b \\ 9c-4d & 18c-9d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -4 & -9 \end{pmatrix}$ ✓ M1

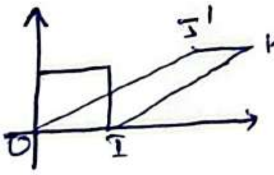
$9a-4b=1$ $\qquad 9c-4d=-4$

$18a-9b=0$ $\qquad 18c-9d=-9$ M1

$a=1,\ b=2$ $\qquad c=0,\ d=1$ ✓

Matrix $N = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ ✓ A1

(ii) Determine the transformation represented by matrix **N**. (2 marks)



Matrix N represent shear x-axis invariant, point $R'(15,-11)$ is transformed to $R''(-7,-11)$ ✓ B1 B1

(c) Find a single matrix that would map triangle PQR onto triangle $P''Q''R''$. (2 marks)

$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix}$ ✓ M1

$= \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix}$ ✓ A1

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