KABOURA JOINT EXAMS

Kenya certificate of secondary education

121/1

Mathematics ALT A

JULY/AUGUST 2023 $2\frac{1}{2}$ HOURS

NAME ....M/S........................ INDEX NUMBER ....

CANDIDATES SIGNATURE .... DATE ....

INSTRUCTIONS TO CANDIDATES

a) Write your name and index number in the spaces provided above
b) Sign and write date of examination in the spaces provided above
c) The paper consists of two sections: Section I and Section II
d) Answer all the questions in section I and only FIVE questions from section II
e) Show all the steps in your calculations, giving your answers at each stage in the spaces provided below each question
f) Marks may be given for correct working even if the answer is wrong
g) Non – programmable silent electronic calculators and KNEC mathematical tables may be used except where stated otherwise.
h) This paper consists of printed pages
i) Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing
j) Candidates should answer the questions in English

FOR EXAMINER'S USE ONLY

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTAL

SECTION II

17	18	19	20	21	22	23	24	TOTAL

GRAND TOTAL

SECTION I (50 MARKS)-*attempt all the questions in this section*

1. Without using a calculator, evaluate (3 marks)

$$\frac{3\frac{1}{2} + \frac{2}{3} \div \frac{4}{9} of\ 2\frac{2}{5}}{2\frac{3}{8}}$$

Numerator.

$\frac{4}{9} \times \frac{12}{5} = \frac{16}{15}$

$\frac{2}{3} \times \frac{15}{16} = \frac{5}{8}$

$\frac{7}{2} + \frac{5}{8} = \frac{28+5}{8} = \frac{33}{8}$ ✓ m1

$\frac{33}{8} \Big/ \frac{19}{8} = \frac{33}{8} \times \frac{8}{19}$ ✓ m1

$= \frac{33}{19}$ ✓ A1

03

2. The L.C.M of three numbers is 900 and their G.C.D is 30. Two of the numbers are 60 and 150. Using factor method, find the least possible value of the third number. (3 marks)

L.C.M = 900 = $2^2 \times 3^2 \times 5^2$

G.C.D = 30 = $2 \times 3 \times 5$

60 = $2^2 \times 3 \times 5$

150 = $2 \times 3 \times 5^2$

m1 (correct factors)

Number = $2 \times 3^2 \times 5$ ✓ m1

= 90 ✓ A1

03.

3. A half of the interior angles of an n sided irregular polygon measures 135^0 each. The remaining half measures 165^0 each. Find n (2 marks)

$(2n-4)90 = \frac{135n}{2} + \frac{165n}{2}$

$180n - 360 = 150n$

$30n = 360$ ✓ m1

$n = 12$ ✓ A1

02

4. Simplify completely

$$\frac{4t^2k-9b^2k^3}{3bk^2-2tk}$$

~~(3 marks)~~ (~~2 marks~~)

$\frac{k(4t^2-9b^2k^2)}{k(3bk-2t)}$ m1

$= \frac{k(\cancel{2t-3bk})(2t+3bk)\times -1}{k(\cancel{3bk-2t})}$ ✓

$= -2t-3bk$ ✓ A1

02

~~10~~

11

5. The mean of five numbers is **20**. The mean of the first three numbers is **16**. The fifth number is greater than the fourth by **8**. Find the fifth number (3 marks)

$20 \times 5 = 100$

$\frac{3 \times 16 + n + n + 8}{5} = 20$ ✓ m1

$48 + 2n + 8 = 100$

$2n = 100 - 56$

$2n = 44$

$n = 22$ ✓ A1

$n + 8 = \cancel{30}\ 22 + 8 = 30$ ✓ B1

6. Solve the inequalities $x \leq 2x + 7 \leq \frac{1}{3}x + 14$. Hence represent the solution on a number line (3 marks)

$x \leq 2x + 7$

$-x \leq 7$

$x \geq -7$ ✓ B1

$2x + 7 \leq \frac{1}{3}x + 14$

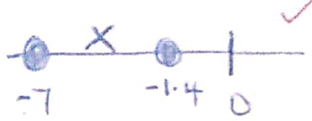
$6x + 21 \leq x + 14$

$\cancel{5x \leq -35}$

$5x \leq -7$

$x \leq -\frac{7}{5}$ ✓ B1

$-7 \leq x \leq -1.4$



✓ B1

7. Given that $4^{3y-4x} = 64$ and $3^y \div 9^x = 1$. solve for x and y (3 marks)

$4^{3y-4x} = 4^3$

$3y - 4x = 3$ ✓

$\frac{3^y}{3^{2x}} = 3^0$

$3^{y-2x} = 3^0$

$y - 2x = 0$ ✓

$y = 2x$

$3y - 4x = 3$

$3(2x) - 4x = 3$

$6x - 4x = 3$

$2x = 3$

$x = 1.5$

$y = 1.5 \times 2$ m1

$y = 3$ m1

A1

8. Use the tables of cubes and reciprocals to evaluate $0.98567^3 + \frac{6}{0.0567}$, giving your answer correct to 2 decimal places (3 marks)

$(9.857 \times 10^{-1})^3 + 6\left(\frac{1}{5.67 \times 10^{-2}}\right)$

$= \left(\frac{955.27}{1000}\right) + 6 \times 100 \times 0.1764$ ✓ m1

$0.955527 + 105.84$ ✓ M1

$= 106.80$ ✓ A1

03

12

9. A wholesaler sold a radio to a retailer making a profit of 20%. The retailer later sold the radio for ksh. 1560 making a profit of 30%. Calculate the amount of money the wholesaler had paid for the radio.(3marks)

$1.3x = 1560$ ✓ M1

$x = 1200$

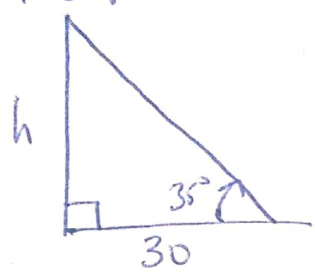
$1.2y = 1200$ ✓ M1

$y = 1000$ ✓ A1

~~Alt~~ Alt

$$\frac{1560}{1.3 \times 1.2} = \underline{1000}$$

10. A pedestrian notice that a tower is at a horizontal distance of 30m away from him and that the angle of elevation of the top of the tower from where he is, is 35°. Find the height of the tower. to 1 d.p. ~~(3marks)~~ (correct ~~(2 marks)~~)



$\tan 35 = \frac{h}{30}$ ✓ ~~M1~~

$h = 30 \tan 35$ ✓ M1

$= 21.0$ ✓ A1 (EAO)

03

11. Find the equation of a straight line which is equidistant from the points (2, 3) and (6, 1) expressing it in the form $ax + by = c$ where a, b and c are constants. (4marks)

mid point

$\frac{2+6,\ 3+1}{2} = (3, 2)$ ✓ B1

Gradient $= -\left(\frac{1-3}{6-2}\right)^{-1}$

$= -\left(\frac{-2}{4}\right)^{-1}$

$= -\left(-\frac{1}{2}\right)^{-1}$

Grad = 2 ✓ B1

$\frac{y-2}{x-3} = 2$ ✓ M1

$2x - 6 = y - 2$

$2x - y = 4$ ✓ A1

4

~~13~~

40

12. Given that the column vectors $a = \begin{pmatrix}1\\2\end{pmatrix}$, $b = \begin{pmatrix}-4\\5\end{pmatrix}$, $c = \begin{pmatrix}3\\-2\end{pmatrix}$ and $p = 2a + b - 3c$. Find $|p|$.

(3marks)

$$p = 2\begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}-4\\5\end{pmatrix} - 3\begin{pmatrix}3\\-2\end{pmatrix}$$

$$p = \begin{pmatrix}2 - 4 - 9\\4 + 5 + 6\end{pmatrix}$$ ✓ m1

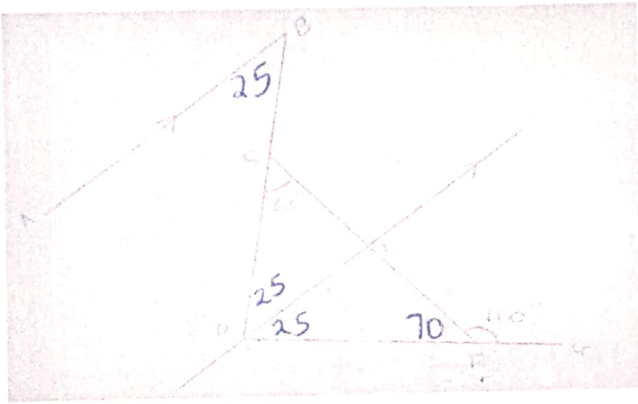
$$p = \begin{pmatrix}-11\\15\end{pmatrix}$$

$$|p| = \sqrt{(-11)^2 + 15^2}$$ ✓ m1

$$= \sqrt{346}$$

$$= 18.60$$ ✓ A1

13. In the figure below, AB is parallel to DE, DE bisects angle BDG, <DCF = 60° and < CFG = 110°.



Calculate the following angles giving reasons in each case.

a) < CDF (2marks)

$180 - 130 = 50°$ ✓ B1

Angles in a triangle add up to 180° ✓ B1

b) < ABD (1marks)

$25°$. alternate angles are equal. ✓ B1

14. A certain amount of money was shared among 3 children in the ratio 7: 5: 3. The largest share was ksh 91. Find.

a) Total amount of money (2marks)

$7 + 5 + 3 = 15$

$\frac{7}{15}x = 91$ ✓ m1

$x = \frac{91 \times 15}{7}$

$= 195$ ✓ A1

b) Difference in the money received as the largest share and the smallest share(2mks)

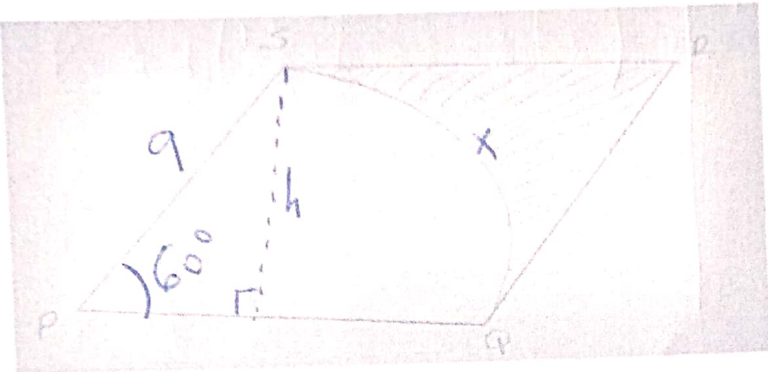
$\frac{3}{15} \times 195 = 39$ ✓ m1

$91 - 39$

$= 52$ ✓ A1

10

15. The figure below shows a rhombus PQRS with PQ = 9 cm and < SPQ = 60°. SXQ is a circular arc, centre P.



Calculate the area of the shaded region correct to two decimal places. (Take $\pi = 3.142$) (3 marks)

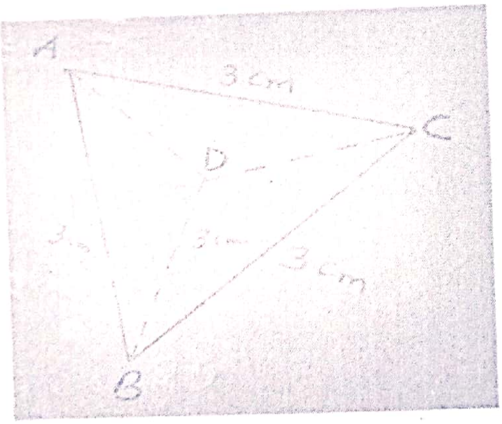
~~(4marks)~~ Area = $Bh - \frac{\theta}{360}\pi r^2$ ✓ M1

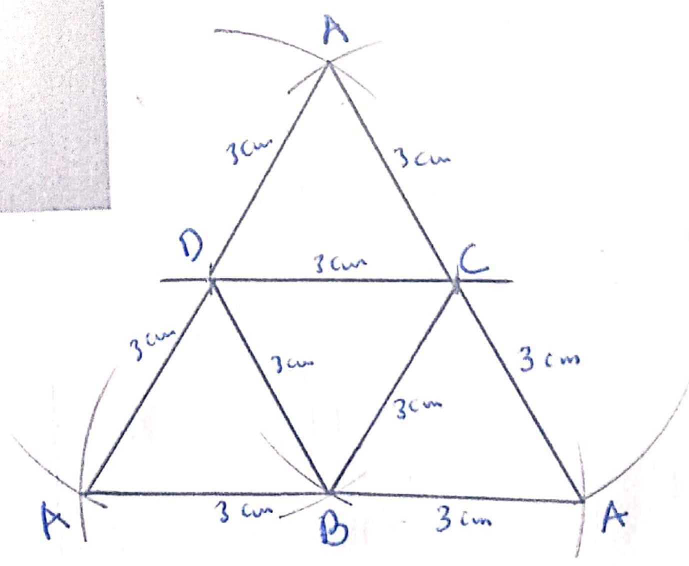
$= (9 \times 9\sin 60) - \frac{60}{360} \times \cancel{\frac{22}{7}}\,3.142 \times 9^2$

$\frac{81\sqrt{3}}{2} - \cancel{\frac{297}{7}}\ 42.417$ ✓ M1

$= 27.73$ ✓ A1

16. Draw a well labelled net of the figure below whose length of each side is 3cm. (3marks)





07

SECTION II (50 MARKS) -*Answer only five questions from this section in the spaces provided*

17. In the year 2021, the price of a table was Ksh 12000.

a) Calculate the amount of money received from the sales of 240 tables that year. (2marks)

240×12000 ✓ M1

$= 2,880,000$ ✓ A1

b) In the year 2022 the price of each table increased by 25% while the number of tables sold decreased by 10%. Calculate the percentage increase in the amount received from the sales. (3marks)

$(12000 \times 1.25) \times (240 \times 0.9) = 3,286,656$ M1

$\frac{3,286,656 - 2,880,000}{2,880,000} \times 100\%$ ✓ M1

$= 14.12\%$ ✓ A1

c) If at the end of the year 2022 the price of each table changed in the ratio 16:15, calculate the price of each table in the year 2023. (1mark)

$15,000 \times \frac{16}{15}$

$= 16,000$ ✓ B1

d) The number of tables expected to be sold by the end of 2023 is ***t*** % less than the number sold in 2021. Calculate the value of ***t*** given that the amounts from the sales in the two years are equal. (4marks)

$\left(\frac{100-t}{100}\right) \times 240 \times 16000 = 2,880,000$ ✓ M1

$2.4(100-t) \times 16000 = 2,880,000$ ✓ M1

$2.4(100-t) = 180$

$100-t = 75$ M1

$t = 100 - 75$ ✓

$t = 25\%$ ✓ A1

10

18. a. Omanyala is practicing for Kip Keino classic competition. He does this in 70 km route from Obambo market to Mbaga Hills. His average speed from Obambo market to Mbaga Hills is *(x + 12)* km/h. He took 20 minutes rest on arrival at Mbaga Hills before running back to Obambo market at an average speed of ***(2x – 4)*** km/h. He took 25 minutes longer from Obambo market to Mbaga Hills than from Mbaga Hills to Obambo market and each of these journeys took him more than one hour.

i) Find the times taken from Obambo market to Mbaga Hills and from Mbaga Hills to Obambo market. (4marks)

$$\frac{70}{x+12} = \frac{70}{2x-4} + \frac{25}{60}$$

$$\frac{70}{x+12} - \frac{70}{2x-4} = \frac{25}{60}$$

m1 A1 B1 B1

$60 \times 70(2x-4) - 70(x+12) \times 60 = 25(x+12)(2x-4)$

$8400x - 16800 - 4200x - 50400 = 25(2x^2 + 24x - 4x - 48)$

$4200x - 67200 = 50x^2 + 500x - 1200$

$50x^2 - 3700x + 66{,}000 = 0$

$x^2 - 74x + 1320 = 0$

$x = 30$ or $x = 44$

$t = \frac{70}{42} = 1\text{ hr } 40\text{ min}$

$t_2 = 1\text{h } 40 - 25$

1hr 15 min

ii) Find Omanyala's average speed to the nearest whole number from the time he left Obambo market for Mbaga Hills to the time he reached Obambo market from Mbaga Hills. (1mark)

Distance = $70 \times 2 = 140$ km

Time taken = 1 hr 40 min + 1 hr 15 min + 20 min

= 3 hrs 15 min = 3.25 hr

Average speed $= \frac{140}{3.25} = 43$ km/h. B1

b. The front of a train whose average speed is ***54km/h*** is at the entry of 120 m long tunnel. It took the train exactly 18seconds to completely get out of the tunnel.

i) Express 54km/h in m/s (2marks)

$54 \times \frac{5}{18} =$ m1

$= 15$ m/s A1

ii) Determine the length of the train (3marks)

Let length of train = x

$D = S \times T$

$\Rightarrow x + 120 = 15\text{ m/s} \times 18\text{ s}$ m1

$x + 120 = 270$

$x = 270 - 120$ m1

$x = 150$ m A1

10

19. a). Find the inverse of $\begin{pmatrix} 2 & 5 \\ 3 & 6 \end{pmatrix}$ (2marks)

$12 - 15 = -3$

$-\frac{1}{3}\begin{pmatrix} 6 & -5 \\ -3 & 2 \end{pmatrix}$ ✓ M1

$= \begin{pmatrix} -2 & 5/3 \\ 1 & -2/3 \end{pmatrix}$ ✓ A1

b). In the month of march, a sales lady sold 8 woolen carpets and 20 door mats at a total of Ksh 34,400. In April of the same year, she sold 12 woolen carpets and 24 door mats at a total of Ksh 48,000. Using matrix method, find the selling price of a woolen carpet and that of a door mat. (4marks)

Let the cost of woolen carpets be x and mats be y ✓ B1 (2 equations formed)

$8x + 20y = 34,400$

$2x + 5y = 8600$ ✓

$12x + 24y = 48000$

$3x + 6y = 12000$ ✓

$\begin{pmatrix} 2 & 5 \\ 3 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8600 \\ 12000 \end{pmatrix}$ ✓ M1 (matrix form)

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & 5/3 \\ 1 & -2/3 \end{pmatrix}\begin{pmatrix} 8600 \\ 12,000 \end{pmatrix}$ M1

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2800 \\ 600 \end{pmatrix}$ ✓ A1

Price, $x = 2800$; $y = 600$ /2

c). At the start of May, she changed the price of a woolen carpet in the ratio 3:4 and reduced the price of a door mat by 6%. In that month she sold twice as many woolen carpets as she sold in the first month and half as many door mats as she sold in the first two months. Use matrix method to find total sales in May. (4marks)

$X_1 = 2800 \times 3/4 = 2100$ } – B1

$y_1 = 600 \times 0.94 = 564$

Carpets sold $= 2 \times 8 = 16$ } – B1

mats $= \frac{1}{2} \times (20 + 24) = 22$

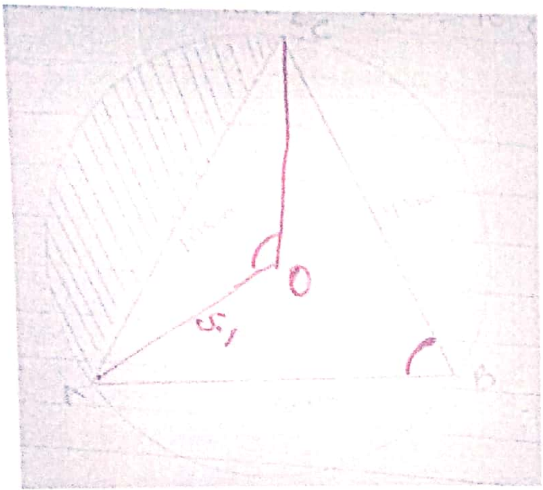
$\begin{pmatrix} 16 & 22 \end{pmatrix}\begin{pmatrix} 2100 \\ 564 \end{pmatrix}$ ✓ M1

$= (2100 \times 16) + (22 \times 564)$ ✓ A1

$= 46,008$

10

20. The figure below (not drawn to scale) shows a triangle ABC inscribed in a circle. AB = 6cm, BC = 9cm and AC = 10cm.



Calculate to 1 decimal place.

i) The angle ABC (3marks)

$10^2 = 6^2 + 9^2 - 2(6\times 9)\cos B$

$\cos B = \frac{10^2 - 6^2 - 9^2}{-108}$ M1

$\cos B = \frac{17}{108}$ M1

$B^\circ = $ ~~9~~ $= 80.9$ A1 C.A.O

ii) The radius of the circle (2marks)

$\frac{10}{\sin 80.9} = 2R$

$R = \frac{5}{\sin 80.9}$ M1

$R = 5.1$ A1

iii) The area of the shaded region. (5marks)

$\theta = 2\times 80.9 = 161.8^\circ$ B1

Area of Sector $= \frac{\theta}{360}\pi r^2$ M1

$= \frac{161.8}{360}\times\frac{22}{7}\times 5.1^2$

$= 36.74\ cm^2$ M1

Area of $\triangle = \frac{1}{2}\times 5.1^2 \sin 161.8$ M1

$= 4.062$

Area of shaded region $= 36.74 - 4.062 = 32.7$ A1

10

21. An object moves along a straight path such that its displacement S meters from a fixed-point O is $S = 2t^3 - 5t^2 + 4t + 2$, where t is time in seconds. Determine

i) The displacement of the object at t = 3 (2marks)

$S = 2(3)^3 - 5(3)^2 + 4(3) + 2$ ✓ m1

$= 23$ ✓ A1

ii) The velocity of the object when t = 3 (3marks)

$V = \frac{ds}{dt}$

$V = 6t^2 - 10t + 4$ ✓ B1

$t = 3$

$V = 6(3)^2 - 10(3) + 4$ ✓ m1

$= 28$ ✓ A1

iii) The times when the velocity of the object is maximum (3marks)

$a = 0$

$\frac{dv}{dt} = 0 = 12t - 10$ ✓ m1

$12t - 10 = 0$ ✓ m1

$12t = 10$

$t = \frac{5}{6}$ ✓ A1

iv) The acceleration of the object when t = 2 (2marks)

$a = 12t - 10$

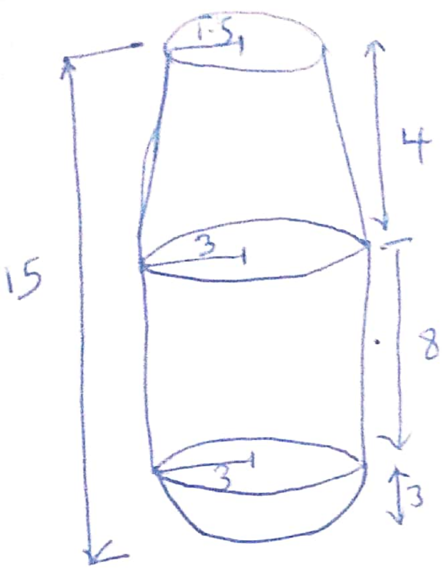
$a = 12(2) - 10$ ✓ m1

$= 14$ ✓ A1

22. The model representing a storage container I s such that the total height of the model is 15cm and is made up of a top that is a frustrum of a cone, a hemispherical bottom and the middle part is cylindrical. The radius of the base of the frustrum of the cone and that of the hemisphere are 3 cm. The top radius of the frustrum is 1.5 cm. The height of the cylindrical part is 8cm.

Calculate

a). the total surface area of the model. (6marks)



S.A of frustrum $= \pi(R+r)L$

$L = \sqrt{(R-r)^2 + h^2}$

$\therefore$ S.A $= \pi(R+r)\sqrt{(R-r)^2+h^2}$

$= \frac{22}{7}(3+1.5)\sqrt{(3-1.5)^2+4^2}$ ✓ M1

$= 60.42\ cm^2$

S.A of cylinder $= 2\pi Rh$

$= 2 \times \frac{22}{7} \times 3 \times 8$ ✓ M1

$= 150\frac{6}{7}\ cm^2$

S.A of hemisphere $= 2\pi r^2$

$= 2 \times \frac{22}{7} \times 3^2$ ✓ M1

$= 56\frac{4}{7}\ cm^2$.

Total S.A $= 60.42 + 150\frac{6}{7} + 56\frac{4}{7}$ ✓ M1 M1

$= 267.8\ cm^2$ (4.s.f) ✓ A1

b). the volume of the model (4marks)

Volume $= \frac{1}{3}\pi h(R^2 + r^2 + Rr) + \pi r^2 h + \frac{2}{3}\pi r^3$

✓ M1 ✓ M1

Any two $= \frac{1}{3} \times \frac{22}{7} \times 4(3^2 + 1.5^2 + 3 \times 1.5) + (\frac{22}{7} \times 3^2 \times 8) + (\frac{2}{3} \times \frac{22}{7} \times 3^3)$

$= 103\frac{5}{7} + 226\frac{2}{7} + 56\frac{4}{7}$ ✓ M1

$= 386\frac{4}{7}\ cm^3$ ✓ A1

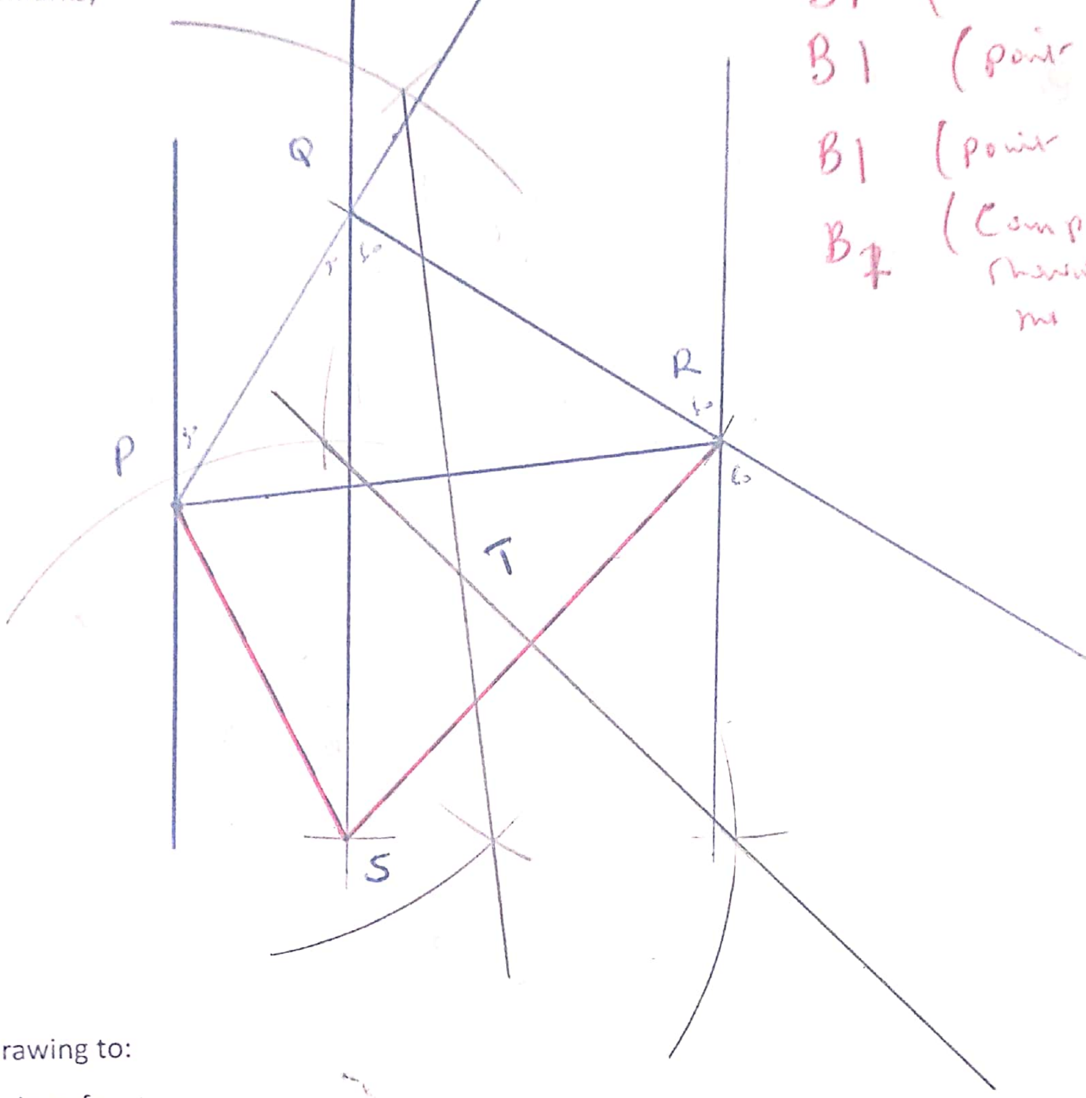
accept 386.6 (4.s.f)

10

23. Three islands **P**,**Q** and **R** are on an ocean such that island Q is 400 km on bearing of 030° from island **P.** Island **R** is 520km and on a bearing of 120° from Island Q. A port **S** is sighted 750 km due south of Island **Q**.

a). Taking a scale of 1cm to represent 100 km, give a scale drawing showing the relative positions of P, Q, R and S. (4marks)

B1 (Correct scale used)
B1 (point Q and R)
B1 (point S located)
B1 (Complete drawing showing all the points)



Use the scale drawing to:

b). find the bearing of:

i) Island R from Island P. (1mark)

$082^\circ \pm 1^\circ$ ✓ B1

ii) Port S from Island R (1mark)

160° ✓ B1

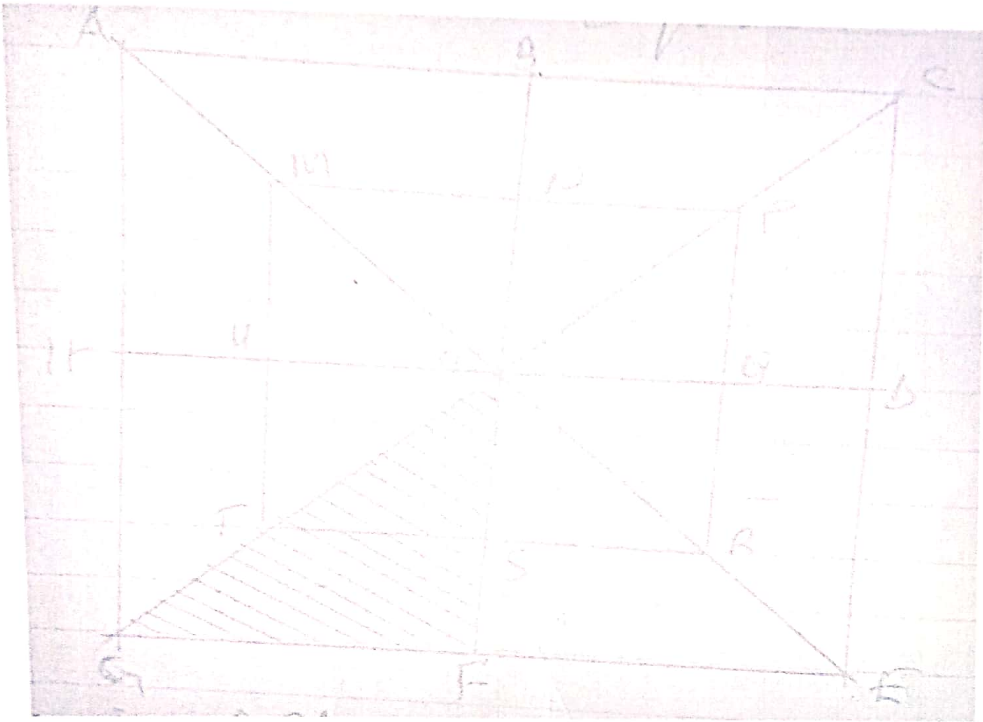
c) Find the distance between Island P and R. (2marks) 6.7×100 ✓ M1 $= 670$ km ± 10 ✓ A1

± 0.1 cm

d) A warship T is such that it is equidistant from Islands P, S and R, by construction locate the position of T. (2marks) ⊥ Bisector of any of the line PR, PS and SR ✓ B1

Locating T and a point of intersection of the two bisectors. ✓ B1

24. The figure below shows two squares ACEG and MPRT. GC and AE are straight lines and O is the common centre of the two squares. Points B,D,F and H are the midpoints of the sides of the larger square while points N,Q,S and U are the midpoints of the smaller square.



a). Describe fully

i). a reflection that maps the shaded part on ΔBCD. (1mark)

Reflection on the line AE B1

ii). a transformation that maps the shaded part onto ΔABO (2marks)

Reflection B1

On the line HD B1

iii). A transformation that maps the shaded part onto Δ NPO (3marks) B1 B1 B1

Enlargement

Scale factor $-\frac{1}{2}$; Centre O

b). Δ PRO is enlarged by a scale factor of +2 with O as the centre of enlargement. The resulting image also undergoes a positive three quarter turn about O. Find the images of P,R and O.

i). after the enlargement (2marks)

C and E respectively B1 B1

ii). after the two successive transformations (2marks)

A ✓ and C ✓ respectively. B1 B1

10