Name..... ADM Number: .....

School: ..... Candidate's Signature.....

121/2
Mathematics Alt. A
FORM FOUR.
JULY, 2023.
2 ½ Hours.

URANGA MATHEMATICS ASSOCIATION 2023

Kenya Certificate of Secondary Education
MATHEMATICS
121/2
FORM FOUR.
TIME: 2 ½ HOURS

INSTRUCTIONS TO CANDIDATES:

- Write your name, school, admission number and sign in the spaces provided above.
- This paper contains **TWO** sections: Section **I** and Section **II**.
- Answer **ALL** the questions in Section **I** and **FIVE** questions from section **II**.
- All answers and working **MUST** be written on the question paper in the spaces provided below each question.
- Marks may be given for correct working even if the answer is wrong.
- Non-programmable silent electronic calculators and KNEC Mathematical tables may be used, except where stated otherwise.

FOR EXAMINERS USE ONLY

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

SECTION II

17	18	19	20	21	22	23	24	Total

Grand Total

This paper consists of 15 printed pages. Candidates should check to ascertain that all pages are printed as indicated and that no questions are missing.

SECTION I (50 MARKS)

Answer ALL the Questions in the spaces provided.

1. A tailor intended to subdivide a piece of cloth into 7 equal parts. She approximated $\frac{1}{7}$ m to 0.14m. Calculate the percentage error in the approximation. (3mks)

Error $\Rightarrow \frac{1}{7} - 0.14$ M1

$\Rightarrow 0.002857$

% Error $\Rightarrow \left(\frac{0.002857}{\frac{1}{7}}\right) \times 100$ M1

$\Rightarrow 2\%$ A1

03

2. Make t the subject of $s = \frac{\sqrt{3d\,(t-d)}}{8}$ (3mks)

$s^2 = \frac{3d(t-d)}{64}$ M1

$64s^2 = 3dt - 3d^2$

$3dt = 64s^2 + 3d^2$ 2M

$t \Rightarrow \frac{64s^2 + 3d^2}{3d}$ A1

OR

$\frac{64s^2}{3d} + d$

03

3. Given that **OA** =3**i** + 4**j** +7**k**, **OB** =4**i** + 3**j** +9**k** and **OC** = **i** + 6**j** + 3**k**. Show that points A, B and C are collinear. (3mks)

A $\begin{pmatrix}3\\4\\7\end{pmatrix}$ B $\begin{pmatrix}4\\3\\9\end{pmatrix}$ C $\begin{pmatrix}1\\6\\3\end{pmatrix}$

$AB \Rightarrow \begin{pmatrix}4\\3\\9\end{pmatrix} - \begin{pmatrix}3\\4\\7\end{pmatrix} \Rightarrow \begin{pmatrix}1\\-1\\2\end{pmatrix}$ M1

$AC \Rightarrow \begin{pmatrix}1\\6\\3\end{pmatrix} - \begin{pmatrix}3\\4\\7\end{pmatrix} \Rightarrow \begin{pmatrix}-2\\2\\-4\end{pmatrix}$ M1

$AC \Rightarrow -2\begin{pmatrix}1\\-1\\2\end{pmatrix}$

$AC = -2AB$, A1

AC is a multiple of AB

AC // AB, common point A. hence Collinear

03

4. Determine the amplitude, period and the phase angle of the curve; (3mks)

$Y=\frac{5}{2}\sin(4x + 60°)$

Amplitude $\Rightarrow$ 2.5 B1

Period $= \frac{360}{4} \Rightarrow 90°$ B1

Phase Angle $\Rightarrow 60°$ B1

03

5. It takes pipe A 8 hours to fill an empty water tank. Pipe B can fill the same tank in 4hours. When the tank is full, pipe C can empty it in 5 hours. Pipes A and B are opened at the same time when the tank is empty. Two hours later, pipe C is opened. Find to the nearest minute the total time taken to fill the tank. (3mks)

1h $\Rightarrow \frac{1}{8}+\frac{1}{4} \Rightarrow \frac{3}{8}$ of the tank

2hr $\Rightarrow 2\times\frac{3}{8} \Rightarrow \frac{3}{4}$ tank B1

Rem $\Rightarrow 1-\frac{3}{4} \Rightarrow \frac{1}{4}$ tank.

1hr $\Rightarrow \frac{1}{8}$ (A) $+\frac{1}{4}$ (B) $-\frac{1}{5}$ (C) $\Rightarrow \frac{7}{40}$ tank

TT $\Rightarrow \frac{1}{4} / \frac{7}{40}$ B1 $\Rightarrow$ 1h 25min.

Total Time Taken

$\Rightarrow$ 2hr + 1hr 25 min

3hrs 25 A1 min

03

6. Given that the expression $4x^2+28x + (k + 37)$ is a perfect square, find the value of k. (3mks)

$\left(\frac{28}{2}\right)^2 = 4(k+37)$ M1

$196 = 4(k+37)$

$49 = k+37$ M1

$k = 12$ A1

03

7. Expand and simplify

$\left(1-\frac{1}{2}x\right)^5$. Hence use your expansion to evaluate $(0.98)^5$ (3mks)

1	1	1	1	1	1	1
$(-\frac{1}{2}x)$	1	$-\frac{1}{2}x$	$\frac{1}{4}x^2$	$-\frac{1}{8}x^3$	$\frac{1}{16}x^4$	$-\frac{1}{32}x^5$
(0.004)	1	5	10	10	5	1

$1-\frac{5}{2}x+\frac{5}{2}x^2-\frac{5}{4}x^3+\frac{5}{16}x^4-\frac{1}{32}x^5$ B1

$1-\frac{1}{2}x=0.98$

$x=0.04$

$1-\frac{5}{2}(0.004)+\frac{5}{2}(0.004)^2-\frac{5}{4}(0.004)^3+\frac{5}{16}(0.004)^4-\frac{1}{32}(0.004)^5$ M1

$\Rightarrow 0.9039$ A1 (03)

8. The price of a piece of land in the year 2010 was Ksh. 700,000. Its value appreciated at the rate of 4% in the first year and then 6% in the subsequent years. Find the price of the same piece of land in the year 2014. (3mks)

$A \Rightarrow P(1+\frac{r}{100})^n$

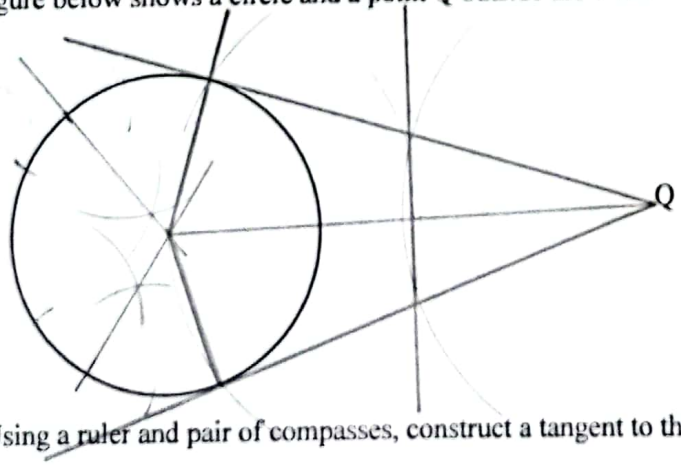
$700000(1+\frac{4}{100})^1$ M1

$728,000$

$Amt \Rightarrow 728000(1+\frac{6}{100})^3$ M1

86705.60 A1 (03)

9. The figure below shows a circle and a point Q outside the circle.



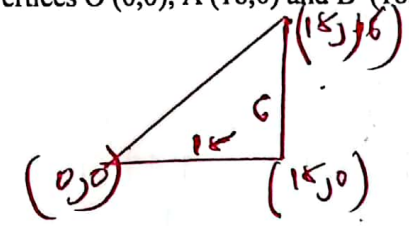
B1 → Centre

B1 → Bisector

B1 Complete diagram. (03)

Using a ruler and pair of compasses, construct a tangent to the circle from Q. (3mrks)

10. Under a transformation $T = \begin{pmatrix} 4 & -3 \\ 2 & 3 \end{pmatrix}$, triangle OAB is mapped onto triangle OA'B' with vertices O (0,0), A'(18,0) and B' (18,6). Find the area of triangle OAB. (3mks)



Object Area $\Rightarrow \frac{1}{2} \times 18 \times 6$

54 u²

det $\Rightarrow 12 - -6 = 18$

$18 = \frac{54}{x}$

$x = \frac{54}{18}$ u

$\Rightarrow 3$ A1

03

11. The series 1 + 4 + 16 + 64 + ... is an increasing geometric progression find the number of terms of the series that would give a sum more than 23,500. (3mks)

$r = \frac{4}{1} = 4$

$S_n \Rightarrow a\left(\frac{r^n - 1}{r - 1}\right)$

$23500 = 1\left(\frac{4^n - 1}{4 - 1}\right)$ u

$23500 = \frac{4^n - 1}{3}$

$70500 = 4^n - 1$

$4^n = 70501$

$n \log 4 = \log 70501$ u

$n = \frac{\log 70501}{\log 4}$

$n = 8.05$

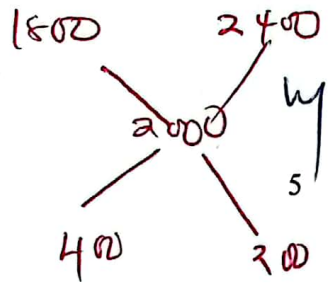
$n = 9$ A1

03

12. A Cocoa blender makes a profit of 30% by selling a super mix cocoa at Ksh. 650 for 250 grams tin. He makes a super mix cocoa by blending two varieties of cocoa, A and B which cost him Ksh. 1,800 and Kshs. 2,400 per kg respectively. In what proportion were the cocoa types A and B be mixed? (3mks)

1kg Mixture $\Rightarrow 650 \times 4 \Rightarrow 2600$

BP $\Rightarrow \frac{100}{130} \times 2600$ u $= 2000$



2 : 1 A1

03

13. A circle whose equation is given as $2x^2 + 2y^2 - 12x + 4ay + 2a^2 - 32 = 0$ passes through a point (3, 1). Find the center and the radius of the circle if 'a' is negative integer. (4mks)

$x^2+y^2-6x+2ay+a^2-16=0$

$3^2+1^2-6(3)+2a(1)+a^2-16=0$ M1

$a^2+2a=24$

$\sqrt{(a+1)^2}=\sqrt{24+1^2}$

$a=4$ or -6 A1

$x^2-6x+y^2-12y=-20$

$(x-3)^2+(y-6)^2=-20+3^2+6^2$ M1

C(3, 6) $r=\sqrt{25}$

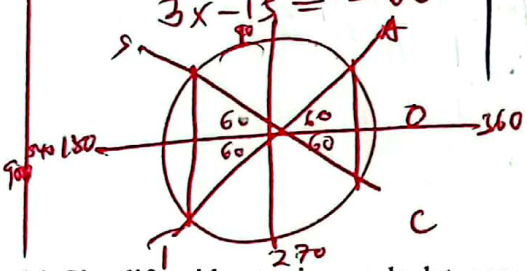
$r=5$ A1

04

14. Given that $2\sin(3x-15)^0 = -\sqrt{3}$ for $0^0 \le x \le 360^0$ find the value of x. (3mks)

$\sin(3x-15)=\frac{-\sqrt{3}}{2}$

$3x-15=-60$ M1



C

$3x-15=240, 300, 600, 660, 960$ M1

$3x=255, 315, 615, 675, 975$

$x=85, 105, 205, 225, 325$ A1 03

14. Simplify without using a calculator or mathematical table; (3mks)

$$\text{Log}_{10}\,200-\frac{1}{3}\log_{10}512+2\log_{10}20$$

$\log\left(\frac{200}{512^{\frac{1}{3}}}\times 20^2\right)$ M1

$\log_{10}10000$

$\log_{10}10^4$

$4\log_{10}10$ M1

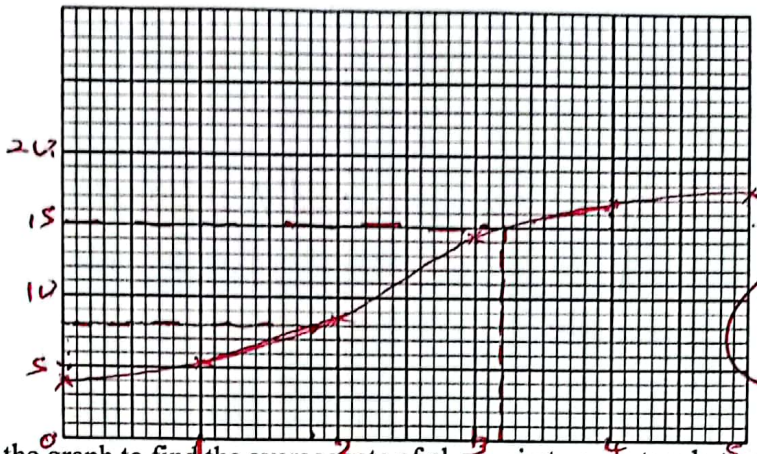
H(1)

$\Rightarrow 4$ A1 03

16. A solution was gently heated, its temperature readings taken at intervals of 1 minute and recorded as shown in the table below:

Time (min)	0	1	2	3	4	5
Temperature (^{o}C)	4	5.2	8.4	14.3	16.8	17.5

a) On the grid below draw a graph of Temperature against Time (2mks)



P
C
1
2

b) Use the graph to find the average rate of change in temperature between t=1.8 and t=3.4. (2mks)

(1.8, 8) (3.4, 15)

grad $\Rightarrow \frac{15-8}{3.4-1.8}$

$\Rightarrow \frac{7}{1.6}$

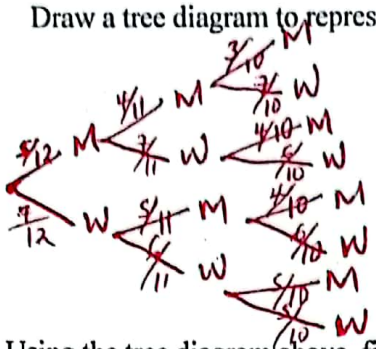
$4\frac{3}{8}$

2

SECTION II(50 Marks)
ANSWER ANY FIVE QUESTIONS

17 a) A wedding committee consisting of three people is to be chosen from five men and seven women.

Draw a tree diagram to represent the above information. (2mks)



Using the tree diagram above, find the probability that:

i). All committee members are of the same gender. (2mks)

$\left(\frac{5}{12}\times\frac{4}{11}\times\frac{3}{10}\right)+\left(\frac{7}{12}\times\frac{6}{11}\times\frac{5}{10}\right)$ M

$\frac{60}{1320}+\frac{210}{1320}$ $\frac{9}{44}$ A

02

ii). At least two of the committee members are men. (2mks)

$\left(\frac{5}{12}\times\frac{4}{11}\times\frac{3}{10}\right)+\left(\frac{5}{12}\times\frac{4}{11}\times\frac{7}{10}\right)+\left(\frac{5}{12}\times\frac{7}{11}\times\frac{4}{10}\right)+\left(\frac{7}{12}\times\frac{5}{11}\times\frac{4}{10}\right)$ M

$\Rightarrow \frac{140}{1320}$

$\Rightarrow \frac{4}{11}$ A

02

b). A tetrahedron is biased such that the probability of a face showing up is given by P (t) = mt where m is a constant and t = 1, 2, 3 and 4 (number of the faces). Find the probability that when the tetrahedron is tossed twice the sum of the faces that will show up is 7. (3mks)

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

M

$\frac{2}{16}$

$\frac{1}{8}$ A

02

18 a). The speed V m/s of a moving particle is partially constant and partially varies as time t seconds. It is given that V = 28 m/s when t = 2 and V = 53 m/s when t = 7 seconds. Find the speed of the particle when t = 11 seconds. (4mks)

$V = K + kt$

$28 = K + 2t$ M

$53 = K + 7t$

$-25 = -5t$ M

$t = 5$

$K = 18$

$V = 18 + 5t$ A1

$= 18 + 5(11)$

$\Rightarrow 73$ B1

04

b). A quantity R varies directly as T and inversely as the cube root of S. Given that S = 64 when T = 6 and R = 30;

i). Find the formula connecting R, S and T. (3mks)

$R = \frac{KT}{\sqrt[3]{S}}$ M

$30 \Rightarrow \frac{6K}{\sqrt[3]{64}}$ M

$K \Rightarrow 20$

$R = \frac{20T}{\sqrt[3]{S}}$ A1

03

ii). Find the percentage change in R when T is decreased by 10% and S increased by 25%. (3mks)

$R' \Rightarrow \frac{96}{100}$

$\frac{}{\sqrt[3]{\frac{125}{100}}}$ M

$\Rightarrow \frac{0.9}{\sqrt[3]{1.25}}$

$\Rightarrow 0.835$

$\frac{(1 - 0.835)}{1} \times 100$ M

$\Rightarrow 16.45\%$ A1

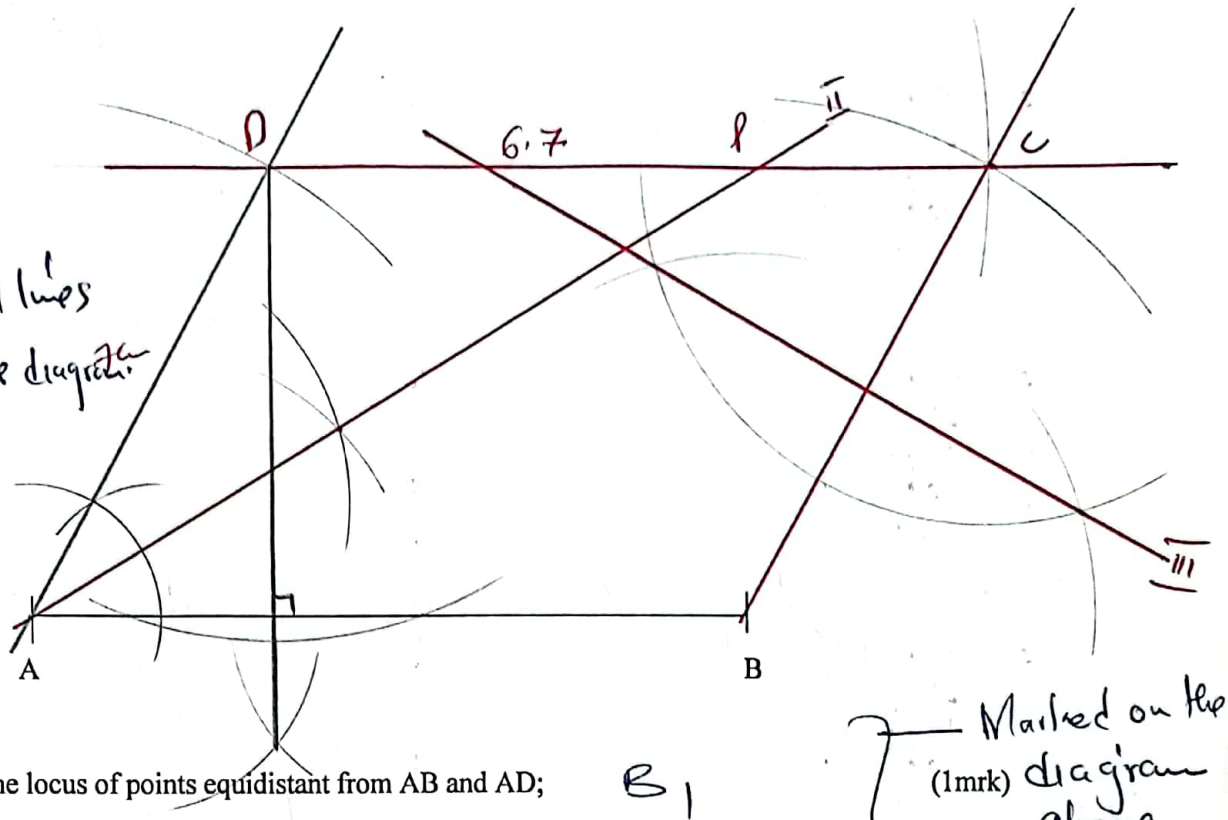
decrease

03

10

19. Using a ruler and a pair of compasses only, construct:

i). A parallelogram ABCD, with line AB below as part of it, such that AD = 7cm and angle BAD = 60^0 (3mks)



B_1 for 60°
B_1 parallel lines
B_1 Complete diagram

ii). The locus of points equidistant from AB and AD; B_1 (1mrk)

ii). The perpendicular bisector BC B_1 (1mrk)

b) i). Mark the point P that lies on DC and is equidistant from AB and AD. B_1 (1mrk)

ii). Measure BP 6cm B_1 (1mrk)

Marked on the diagram above.

c). Describe the locus that the perpendicular bisector of BC represents. (1mrk)

Line bisector of line BC in the ratio 1:1 B_1 or any equivalent

d). Calculate the area of trapezium ABCP. (2mrks)

$(10.3 \times 6) - (\frac{1}{2} \times 6 \times 6.7 \sin 120^\circ)$ M

$\Rightarrow 61.8 - 20.38$

$\Rightarrow 41.49\ cm^2$. A

02

10

20. The data below shows the masses in kg of 50 calves in a dairy farm.

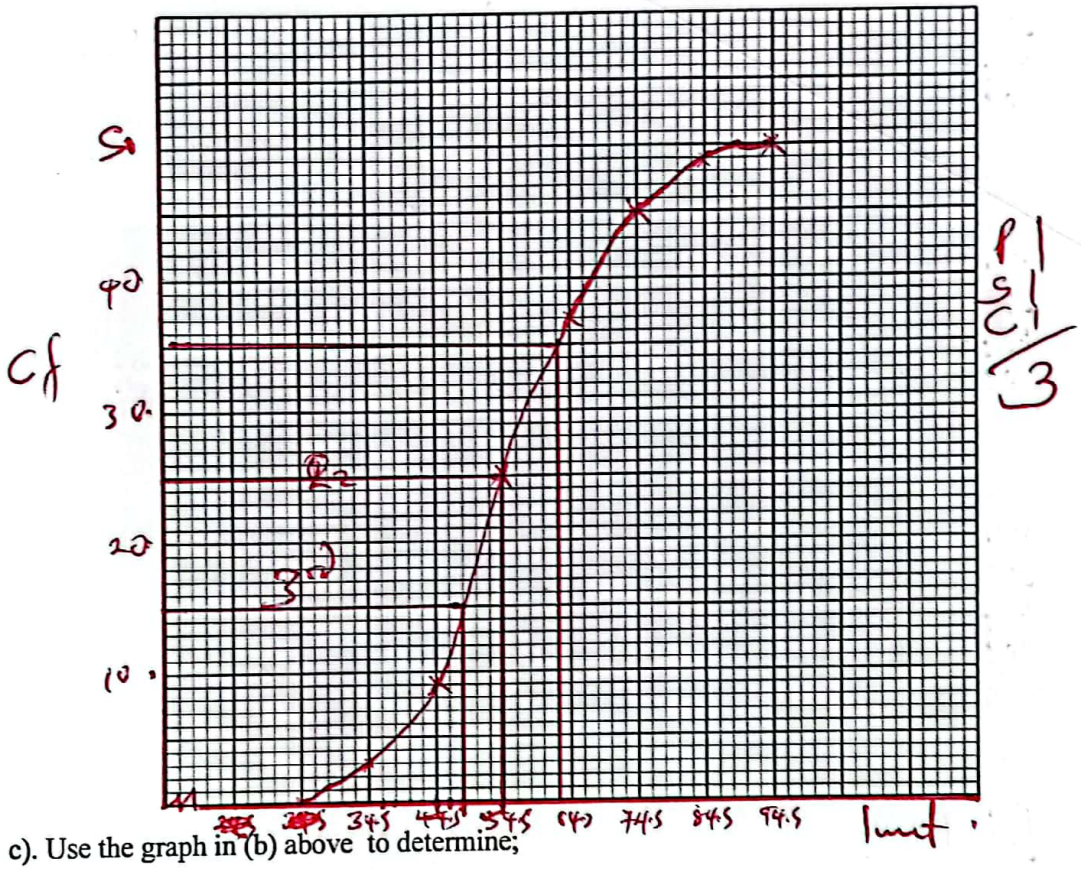
Mass (kg)	25 - 34	35 - 44	45 - 54	55 - 64	65 - 74	75 - 84	85 - 94
No. of calves	X	2X	16	4X	8	4	1
	3	9	25	37	45	49	50

a). Determine the value of X (2mrks)

$7x = 50 - 21$ M1 $x = 3$ A1

02

b). On the grid provided draw a cumulative frequency curve for the data (4mrks)



P1
S1
C1
3

c). Use the graph in (b) above to determine;

i). The median (1mrk)

54.5 B1

01

ii). The difference between the 3rd decile and the 7th decile (3mrks)

3rd decile $\Rightarrow \frac{3}{10} \times 50^{th} = 15^{th} \Rightarrow 48.5$ kg M1

7th decile $\Rightarrow \frac{7}{10} \times 50^{th} \Rightarrow 35^{th} \Rightarrow 62.5$ kg

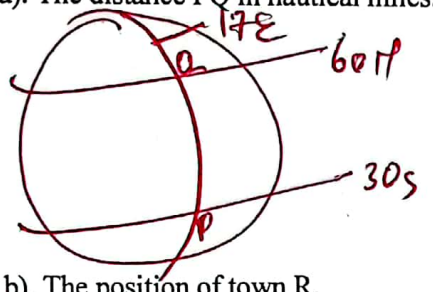
$(62.5 - 48.5) \Rightarrow 14$ A1

03

10

21. An aircraft leaves town P(30^0 S, 17^0 E) and flies due north to Q (60^0N, 17^0 E). It then flies at an average speed of 300 knots for 8 hours due West to town R. Determine;

a). The distance PQ in nautical miles. (2mks)



Dis $\Rightarrow \theta \times 60$
$\Rightarrow 90 \times 60$ M
$\Rightarrow 5400$ nm A1 (02)

b). The position of town R. (4mks)

dis $\Rightarrow 300 \times 8 \overset{M}{\Rightarrow} 2400$nm

$D = \theta \times 60\cos\alpha$
$2400 = \theta \times 60\cos 60$ M
$\theta \Rightarrow 80$ A1

$80 - 17 = 63$

R(60N, 63W) B1 (04)

c). The local time at R if the local time at Q is 3.12 pm. (2mks)

TT $\Rightarrow \frac{80 \times 4}{60}$ M
$\Rightarrow$ 5hrs 20min M

15 12
5 20
0952
9:52Am A1 (02)

d). The distance travelled by the aircraft from Q to R to the nearest kilometer.

($\pi = \frac{22}{7}$ and $R = 6370km$) (2mks)

$dis = \frac{\theta}{360} 2\pi R\cos\alpha$

$\frac{80}{360} \times 2 \times \frac{22}{7} \times 6370\cos 60$ M

4449km A1 (02)

22. A trader stocks two brands of rice A and B. The rice is packed in packets of the same size. The trader intends to order fresh supplies but his store can accommodate a maximum of 500 packets. He orders at least twice as many packets of A as of B. He requires at least 50 packets of B and more than 250 packets of A. If he orders X packets of A and Y packets of B.

a). Write the inequalities in terms of X and Y which satisfy the above information. (4mks)

i) $x + y \leq 500$ B1

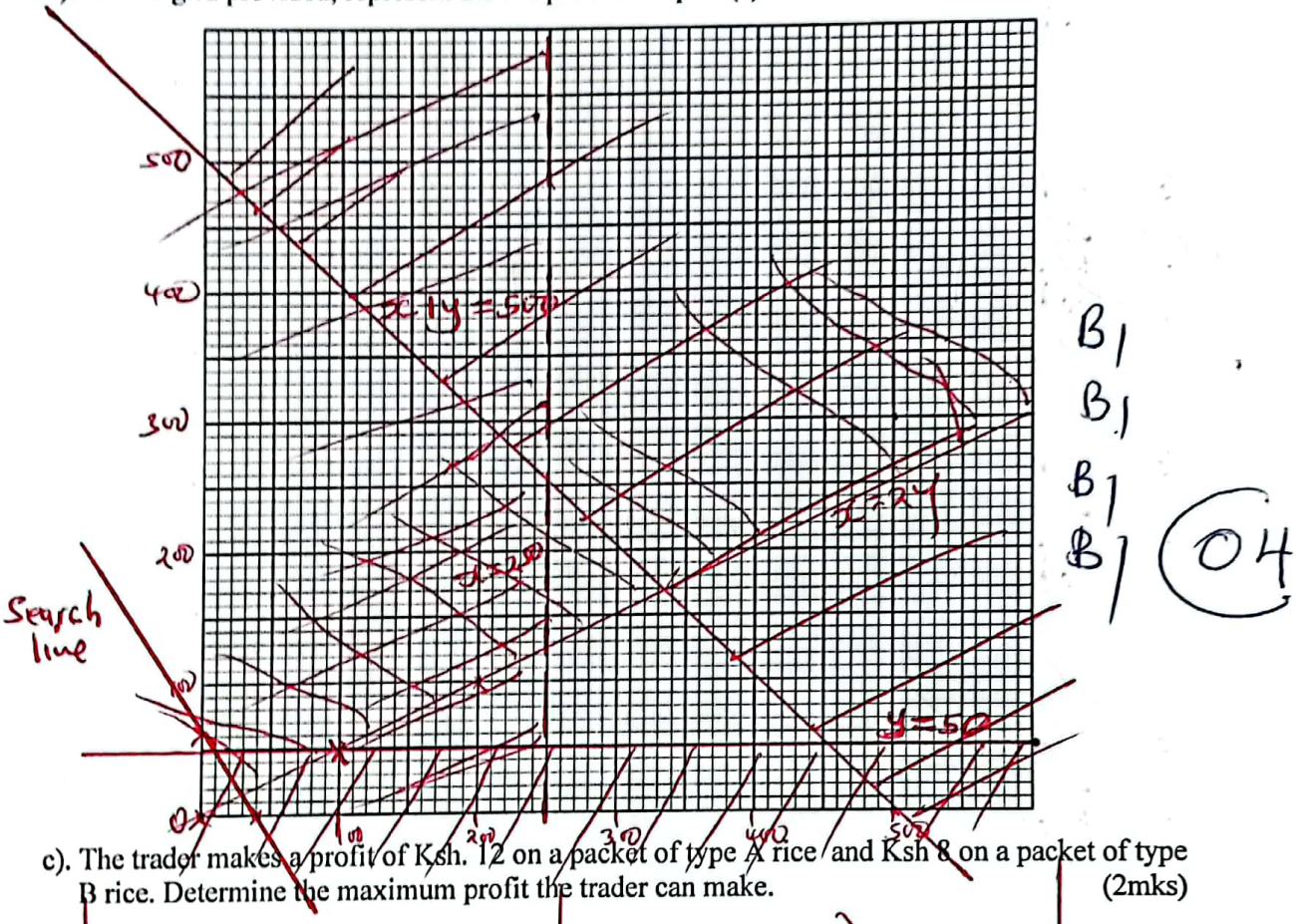
ii) $x \geq 2y$ B1

iii) $y \geq 50$

$x > 250$ B1

04

b). On the grid provided, represent the inequalities in part (a) above. (4mks)



B1
B1
B1
B1
04

c). The trader makes a profit of Ksh. 12 on a packet of type A rice and Ksh 8 on a packet of type B rice. Determine the maximum profit the trader can make. (2mks)

$12x + 8y = 480$

$3x + 2y = 120$

x	0	40
y	60	0

(450, 50)

$12(450) + 8(50)$

$5400 + 400$

5800 A1

02

23. A triangle A′BC′ with vertices at A (1, -1), B (3, -1) and C (1, 3) is mapped onto triangle A′B′C′ by a transformation whose matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

Triangle A′B′C′ is mapped onto A″B″C″ with vertices at A″ (2,2), B″ (6,2) and C″ (2,-6) by a second transformation.

a). Find the coordinates of A′B′C′ (2mks)

$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{matrix} A & B & C \end{matrix}\begin{pmatrix} 1 & 3 & 1 \\ -1 & -1 & 3 \end{pmatrix} = \begin{matrix} A' & B' & C' \end{matrix}\begin{pmatrix} -1 & -3 & -1 \\ -1 & -1 & 3 \end{pmatrix}$ M1

A′(-1,-1) B′(-3,-1) C′(-1,3) A1

02

b). Find the matrix which map A′B′C′ onto A″B″C″ (3mrks)

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{matrix} A' & B' & C' \end{matrix}\begin{pmatrix} -1 & -3 & -1 \\ -1 & -1 & 3 \end{pmatrix} = \begin{matrix} A'' & B'' & C'' \end{matrix}\begin{pmatrix} 2 & 6 & 2 \\ 2 & 2 & -6 \end{pmatrix}$ M1

$-a-b=2$
$-3a-b=6$
$2a=-4$
$a=-2, b=0$

$-c-d=2$
$-3c-d=2$
$2c=0$ M1
$c=0$
$d=-2$

$\begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}$ A1

03

c). Determine the ratio of the area of triangle ABC to triangle A″B″C″ (3mrks)

$\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$ M1

$\det \Rightarrow +4-0$
$\Rightarrow -4$

$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = (+4) = -1$ M1

Ratio
$-1 : -4$
$1 : 4$ A1

03

d). Find the transformation matrix which maps A″B″C″ onto ABC (2mrks)

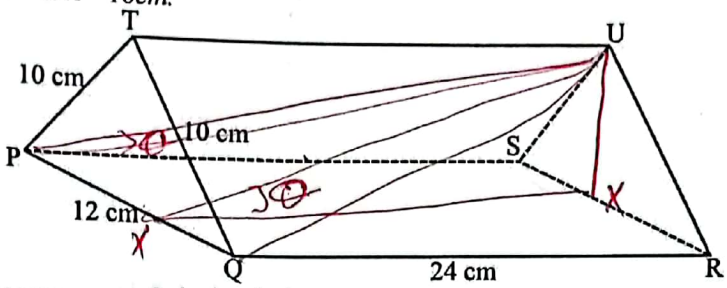
$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$ M1

$\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$ A1

02

10

24. The figure below shows a triangular prism PQRSTU in which PQ = 12*cm* , QR = 24*cm* and TP = PQ = US = UR = 10*cm*.



Leaving your answers to 2 decimal places:

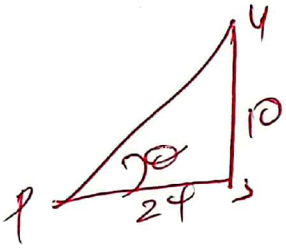
(a) Find the length:

i) PR $\sqrt{24^2+12^2} = 26.83$ B1 (01) (1 mark)

ii) PU $\sqrt{24^2+10^2} = 26.00$ B1 (01) (1 mark)

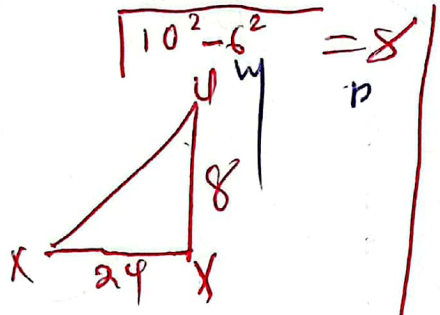
(b) Calculate:

i) The angle between the line PU and the base PQRS. (3 marks)



$\tan\theta = \frac{10}{24}$ M1 M1 $\Rightarrow 22.62^\circ$ A1 (03)

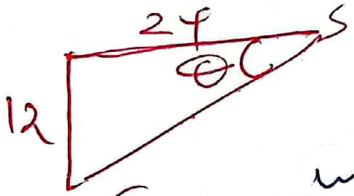
ii) The angle between the planes UPQ and the base PQRS. (3 marks)



$\sqrt{10^2-6^2} = 8$ P

$\tan\theta = \frac{8}{24}$ M1 $\theta \Rightarrow 18.43^\circ$ A1 (03)

(c) Calculate the size of the acute angle between the lines TU and QS. (2 marks)



$\tan\theta = \frac{12}{24}$ M1 15

$\theta \Rightarrow 26.57^\circ$ A1 (02)

10