URANGA MATHEMATICS ASSOCIATION 2023
TERM ONE FORM FOUR PP2 MARKING SCHEME.

SECTION 1 (50marks)
(Answer ALL questions in this section)

1. The length and width of a rectangle were measured as 12.4cm and 5.0cm respectively. Find to 4 significant figures, the percentage error in calculating the area of the rectangle (3 marks)

Max A = 12.45 × 5.05 = 62.875
True A = 12.4 × 5.0 = 62.00 B1
Min A = 12.35 × 4.95 = 61.1325

$A_e = \frac{62.875 - 61.1325}{2} = 0.87$

$\%e = \frac{0.87}{62} \times 100$ M1

$= 1.4032 = 1.403\%$ A1 cao

ALT.

$Re = \left(\frac{0.05}{12.4} + \frac{0.05}{5.0}\right)$ M1 $= \frac{87}{6200} = 0.01403$

$\%e = 0.01403 \times 100$ M1

$= 1.403\%$ A1.

2. The co-ordinates of a point A is (2, 8, 3) and B is (-4, -8, -5). A point P divides AB externally in the ratio 7: 3. Determine the co-ordinates of P (3 marks)

7 : -3.

$OP = \frac{7}{4}\begin{pmatrix}-4\\-8\\-5\end{pmatrix} + \frac{-3}{4}\begin{pmatrix}2\\8\\3\end{pmatrix}$ M1

$= \begin{pmatrix}-7\\-14\\-\frac{35}{4}\end{pmatrix} - \begin{pmatrix}\frac{3}{2}\\-6\\-\frac{9}{4}\end{pmatrix} = \begin{pmatrix}-8\frac{1}{2}\\-20\\-11\end{pmatrix}$ A1

$P(-8\frac{1}{2}, -20, -11)$ B1

3. Solve for x: $(\log_3 x)^2 - \frac{1}{2}\log_3 x = \frac{3}{2}$ (3 marks)

let $\log_3 x = t$.

$t^2 - \frac{1}{2}t = \frac{3}{2}$.

$2t^2 - t - 3 = 0$ M1

$t = \frac{1 \pm \sqrt{1+24}}{4}$

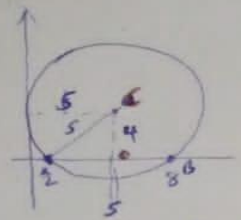
$= \frac{1 \pm 5}{4}$

$t = \frac{3}{2}, t = -1$ A1 both ✓

$x = 3^{\frac{3}{2}}$ or $x = 3^{-1}$

$x = 5.196$ $x = \frac{1}{3}$ B1 both ✓.

A circle is tangent to the y – axis and intersects the x- axis at (2,0) and (8,0). Obtain the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are integers (4 marks)



Mid-chord = (5,0)
⇒ r = 5.
AB = 3 units.
⇒ AC = 4 units.
⇒ C(5,4) B1

$(x-5)^2 + (y-4)^2 = 5^2$ M1
$x^2 - 10x + 25 + y^2 - 8y + 16 = 25$
$x^2 + y^2 - 10x - 8y + 16 = 0$. A1.

Solve for X in the equation:
$2\sin^2 x - 1 = \cos^2 x + \sin x$, for $0^0 \leq x \leq 360^0$ (4 marks)

$2\sin^2 x - 1 = 1 - \sin^2 x + \sin x$.
$3\sin^2 x - \sin x - 2 = 0$. M1
Let $\sin x = a$
$3a^2 - a - 2 = 0$.
$(3a+2)(a-1) = 0$. M1
$a = -2/3$. $a = 1$ A1.

$\sin x = -2/3$.
$\sin^{-1}(2/3) = 41.81°$, $x = 221.81°$, $318.19°$
$x = 1$
$\sin^{-1} 1 = 90$, $x = 90$ B1✓ 3 orders✓

Make P the subject of the formula $T = \sqrt[3]{\frac{p^2+n}{m^2}} + R$ (3 marks)

$T - R = \sqrt[3]{\frac{p^2+n}{m^2}}$

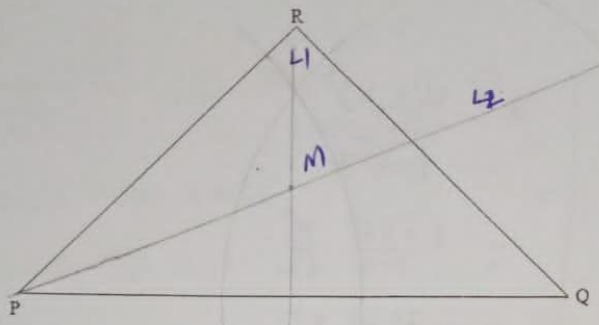
$(T-R)^3 = \frac{p^2+n}{m^2}$ M1

$m^2(T-R)^3 = p^2 + n$.

$m^2(T-R)^3 - n = p^2$ M1

$P = \pm\sqrt{m^2(T-R)^3 - n}$ A1

Below is a triangle PQR. Draw locus L1 of points equidistant from P and Q. Draw L2 locus of points equidistant from line PR and PQ to meet L1 at M. Measure PM. (3 marks)

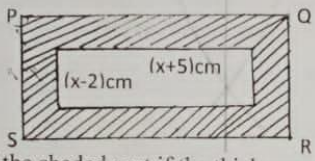


L_1 ✓ B_1

L_2 ✓ B_1

$PM = 5.1$ $(5-5.2)$ B_1

8. PQRS is a rectangle whose area 170cm^2. The internal rectangle measures $(x+5)cm$ by $(x-2)cm$.



Determine the area of the shaded part if the thickness of this part is $0.1x\ cm$ (4 marks)

$(x+5+0.2x)(x-2+0.2x) = 170$ M_1

$(1.2x+5)(1.2x-2) = 170$.

$1.44x^2 + 3.6x - 180 = 0$.

$2x^2 + 5x^2 - 250 = 0$.

$x = \dfrac{5 \pm \sqrt{25+2000}}{4}$ M_1

$\dfrac{5+45}{4} = -12.5$, ignore or 10

$(10-2)(10-5)$ M_1

$= 8 \times 15 = 120$

Shaded area

$= 170 - 120$

$= 50\ cm^2$ A_1

9. Find the length of an arc of a circle which subtends an angle of 0·8 radians at the center of the circle. The radius of the circle is 15 cm. (2 marks)

$1^c = 15cm$.

$0.8^c =$

$\dfrac{0.8 \times 15}{1} = 12\ cm$. M_1 A_1.

10. A dealer has two types of grades of tea, A and B. Grade A costs shs.140 per kg while grade B costs shs.160 per kg. Determine the ratio he should mix A and B so that he makes a 60% profit when he sells the mixture at shs.232 per kg. (3 marks)

$\frac{140A + 160B}{A+B} = 145$ B1 B1

$140A + 160B = 145A + 145B$

$15B = 5A$ B1

$\frac{A}{B} = \frac{15}{5}$

$A:B = 3:1$ A1

ALT.

A	B
140	160
	145 B1
15	5 B1

$15:5$

$= 3:1$ B1

✓ Mark mis B1

11. The data below shows marks scored by 8 form four students in Molo district mathematics contest 44, 32, 67, 52, 28, 39, 46, 64. Calculate the mean absolute deviation of the data. (3 marks)

$\bar{x} = \frac{44+32+67+\ldots+64}{8} = 46.5$ B1

x	44	32	67	52	28	39	46	64
d	-2.5	-14.5	20.5	5.5	-18.5	-7.5	-0.5	17.5

$|\bar{d}| = \frac{|-2.5|+\ldots+17.5}{8} = 10.875$ M1 A1

12. variable y varies as the square of x and inversely as the square root of z. Find the percentage change in y hen x is changed in the ratio 5:4 and z reduced by 19% (3 marks)

$y = \frac{Kx^2}{\sqrt{z}}$ M1

$y' = \frac{K\left(\frac{5}{4}x\right)^2}{\sqrt{0.81z}}$

$y' = \frac{\frac{25}{16}Kx^2}{0.9\sqrt{z}}$ M1

$= 1.7361\frac{x^2}{\sqrt{z}}$ MA1

$(1.7361 - 1) \times 100\%$ MA1

$= 73.61\%$ increase. A1

13. Simplify $\frac{2\sqrt{2}}{1+\sqrt{2}} - \frac{\sqrt{2}}{1-\sqrt{2}} = a + b\sqrt{c}$ leaving your answer in the form $a + b\sqrt{c}$, where a, b and c are rational numbers.

(3 marks)

$$\frac{2\sqrt{2}(1-\sqrt{2}) - \sqrt{2}(1+\sqrt{2})}{1-2} \quad M_1$$

$$= \frac{2\sqrt{2} - 4 - \sqrt{2} - 2}{-1} \quad M_1$$

$$= \frac{\sqrt{2} - 6}{-1}$$

$$= 6 - \sqrt{2}. \quad A_1$$

14. Find the value of x given that T is a singular matrix. (3 marks)

$$T = \begin{bmatrix} x+1 & 2 \\ 4x & 2x \end{bmatrix}$$

$$2x(x+1) - 8x = 0. \quad M_1$$

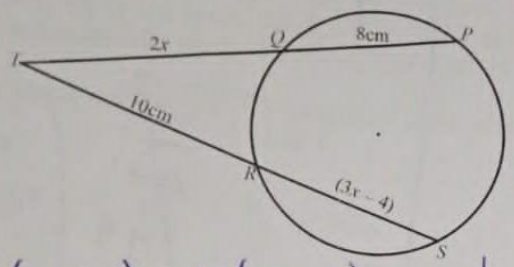
$$2x^2 + 2x - 8x = 0$$

$$2x^2 - 6x = 0.$$

$$2x(x-3) = 0 \quad M_1$$

$x = 0$ } or $x = 3$. } A_1 both ✓.

15. The figure below PQT and SRT are ~~chords~~ secants which are intersecting at ~~T of a circle~~. $PQ = 8cm$, $QT = 2x$ cm, $SR = (3x - 4)$ and $RT = 10cm$. Find the value of x ~~to~~ 2 sf (3 marks)



$2x(2x+8) = 10(3x+6)$. M1

$4x^2 + 16x = 30x + 60$.

~~$4x^2 - 14x - 60$~~

$4x^2 - 14x - 60 = 0$

$2x^2 - 7x - 30 = 0$. ~~A1~~

$x = \frac{7 \pm \sqrt{49 + 240}}{4}$

$= \frac{7 \pm 17}{4}$

$= 6$ or -2.5 (ignore) } A1.

$x = 6$ cm.

6.0 B1 CAO.

16. Expand $(x - y)^6$. Hence use the first four terms to estimate the value of 1.98^6. Giving your answer correct to four decimal places. (3 marks)

$1.x^6y^0 - 6x^5y + 15.x^4y^2 - 20x^3y^3 + 15.x^2y^4 - 6xy^5 + 1.x^0y^6$

$x^6 - 6x^5y + 15x^4y^2 - 20x^3y^3 + 15x^2y^4 - 6xy^5 + y^6$. M1

$x - y = 1.98$. let $x = 2$

$2 - y = 1.98$

$\Rightarrow y = 0.02$

$2^6 - 6(2)^5(0.02) + 15(2)^4(0.02)^2 - 20(2)^3(0.02)^3$ M1

$64 - (6)(32)(0.02) + (240)(0.02)^2 - (160)(0.02)^3$.

$= $ ~~1,283.9347~~

$\simeq$ ~~1,283.9347~~.

$64 - 3.84 + 0.096 - 0.00128$

$= 60.2547$ A1 CAO.

SECTION II 50 marks

(Answer ONLY FIVE questions in this Section)

17. A tank has two water taps P and Q and another tap R. When empty the tank can be filled by tap P alone in 5 hours or by tap Q in 3 hours. When full the tank can be emptied in 8 hours by tap R.

a) The tank is initially empty. Find how long it would take to fill up the tank;

i) If tap R is closed and taps P and Q are opened at the same time. (2 marks)

In 1hr.

P & Q fill $\frac{1}{5}+\frac{1}{3}=\frac{8}{15}$ of tank

Whole tank → $1\times1\times\frac{15}{8}$ M_1 = 1hr, 52 min 32 sec (accept 1.875 hrs) A_1.

ii) If all the three taps are opened at the same time. Giving your answer to the nearest minutes. (2 marks)

P Q & R ⇒ $\frac{8}{15}-\frac{1}{8}=\frac{49}{120}$.

$1\times1\times\frac{120}{49}$ M_1

= 2 hr, 26 min, 56 sec.

≈ 2 hr 27 min A_1.

Accept 147 min.

b) Assume the tank initially empty and the three taps are opened as follows;
P at 8:00 am, Q at 9:00 am and R at 9:00 am

i) Find the fraction of the tank that would be filled by 10:00 am. (3 marks)

fraction filled by P = $\frac{1}{5}$.

P Q R = $\frac{49}{120}+\frac{1}{5}=\frac{73}{120}$ M_1 A_1.

ii) Find the time the tank would be fully filled up. Give your answer to the nearest minute. (4 marks)

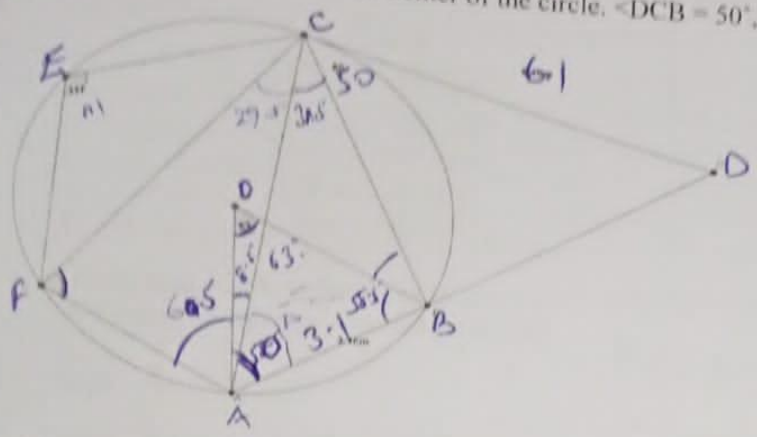
at 10 am $\frac{73}{120}$ of Remaining $1-\frac{73}{120}=\frac{47}{120}$. B_1

1 hr = $\frac{49}{120}$

$\frac{47}{120}\times1\times\frac{120}{49}$ M_1 = 57.55 min ≈ 58 min A_1

10.00 + 58 = 10:58 B_1.

8. In the figure below, G is the center of the circle. $\angle DCB = 50^\circ$, $\angle AGB = 63^\circ$ and $\angle FEC = 111^\circ$



(a)Determine the size of the following angles

i. FAC $180 - 111 = 69^\circ$ B1 (1 mark)

ii. BAC $DCB(50^\circ) = CAB = 50^\circ$ B1 (1 mark)

iii. ACB $31\frac{1}{2}^\circ$ B1 (1 mark)

iv. ACF $FAB = 119^\circ$, $FCB = 61$, $ACF = 61 - 31.5 = 29.5^\circ$ B1 (1 mark)

v. AFC $180 - 31.5 = 149.5^\circ - 50 = 98.5^\circ$, $AFC = 180 - 98.5 = 81.5^\circ$ B1 (1 mark)

(b)DC is a tangent to the circle at C and its length is 6.1cm. AD is a straight line and chord AB is 3.1cm. Determine

(i) The length of BD

$(3.1 + x)x = 6.1^2$ M1

$x^2 + 3.1x - 37.21 = 0$

$x = \frac{-3.1 \pm \sqrt{9.61 + 148.84}}{2}$

$x = -7.845$ ignore

$x = 4.745$

$BD = 4.745$ cm. A1 (2 marks)

(ii) Area of triangle ABC

$\frac{3.1}{\sin 31.5} = \frac{BC}{\sin 50}$ M1

$BC = \frac{3.1}{\sin 31.5} \cdot \sin 50$ A1

$= 4.545$

$A = \frac{1}{2} \times 4.545 \times 3.1 \sin 98.5$ M1

$= 6.967$ A1. (3 marks)

7. The table below shows the income tax rates in a certain year.

Annual taxable income in Kenya shillings	Tax rate in %
0-144000	0%
144 001 -300 000	10
300 001 – 468 000	15
468 001 -648 000	20
648 001-840 000	25
Above 840 000	30

During that year Kurenta's annual gross tax in sixth band was Shs 108 000.

(a) Determine Kurenta's annual gross tax. (3marks)

Band 1 $144,000 \times 0\% = 0$

slab 2 $156\,000 \times 10\% = 15,600$

slab 3 $168\,000 \times 15\% = 25,200$

slab 4 $150,000 \times 20\% = 36,000$

slab 5 $192\,000 \times 25\% = 48,000$

slab 6 $S \times 30\% = 108\,000$

$S = 360,000/=$

M_1 Gross tax $= 15600 + 25,200 + 36,000 + 48,000 + 108,000$

$= 232,800$ A_1.

(b) If he enjoyed annual relief of Shs. 21 000, determine his monthly net tax (P.A.Y.E) (2 marks)

$$\frac{232500 - 21,000}{12} \quad M_1$$

$= 17650$ A_1.

(c) Kurenta had a basic salary of Shs X Shs p.a. and enjoyed non-taxable allowances that is equivalent to 45% of basic salary. Determine Kurenta's gross salary p.m. (3 marks)

Slab six (TI) $\Rightarrow 108000 \div 30\%$

$= 360,000/=$ B_1

$TI = 360,000 + 840,000$

$= 1,200,000$ B_1

$\frac{145}{100} \times 1,200,000 = 1,740,000$

$\frac{1,740,000}{12}$

$= 145,000/=$ B_1.

(d) The following deductions were also made from Kurenta's salary every month: Co-operative shares Sh.8 000, Co-operative loans Sh. 12 000, Pension scheme Sh.4 000, Union dues Sh. 2 000. Determine Kurenta's monthly net salary during that year. (2 marks)

Net salary $145,000 - (17650 + 8000 + 12000 + 4000 + 2000)$ M_1

$= 101,350$ A_1.

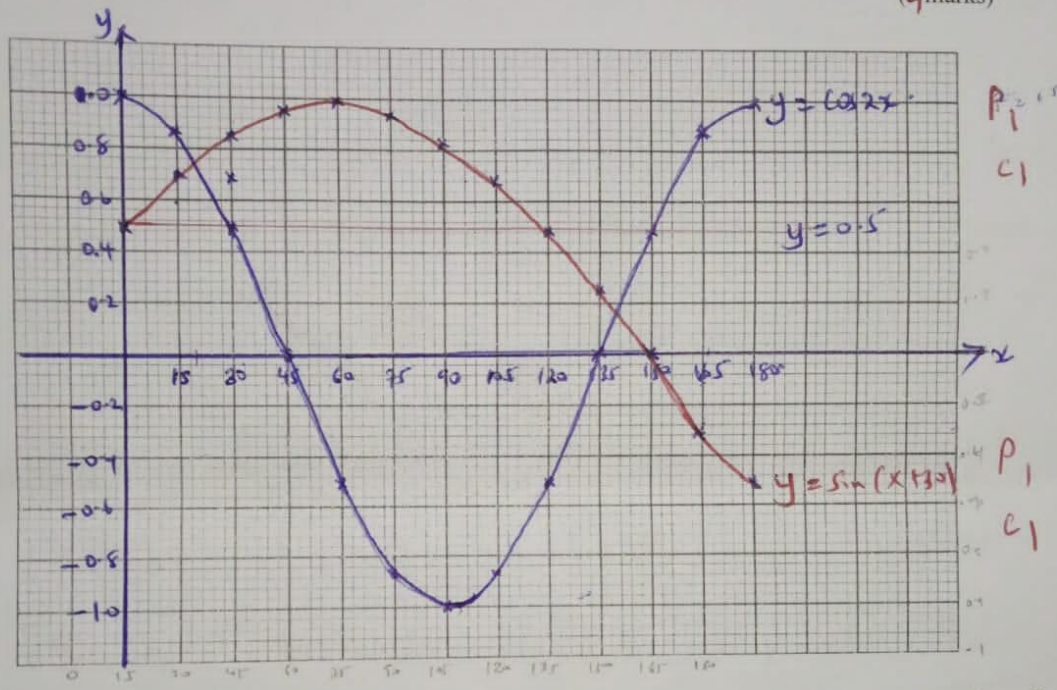
Given that $y = \cos 2x^0$ and $y = sin(x^0 + 30^0)$

(i) Complete the table below, giving the values correct to 2 decimal places (2 marks)

X^0	0^0	15^0	30^0	45^0	60^0	75^0	90^0	105^0	120^0	135^0	150^0	165^0	180^0
$\cos 2X^0$	1.00	0.87	0.50	0.00	-0.5	-0.87	-1.00	-0.87	-0.5	0.00	0.50	0.87	1.00
$\sin (X^0+30^0)$	0.50	0.71	0.87	0.97	1.00	0.97	0.87	0.71	0.50	0.26	0.00	-0.26	-0.50

B1 all. B1 ✓all

ii). Using the grid provided draw on the same axes the graph of $y = \cos 2x^0$ and $y = sin(x^0 + 30^0)$ *for* $0^0 \leq X \leq 180^0$. (4 marks)



P1 C1 P1 C1

ii) Find the period of the curve $y=cos(2x^0)$ (1 mark)

180° B1

iii) Using the graph, estimate the solutions to the equations;

19.5° ± 1° , 139.5° ± 1° B1 B1

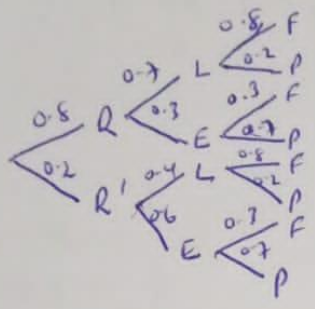
a). $sin(x^0 + 30^0) = cos(2x^0)$ (2 mark)

b). $cos(2x^0) = 0.5$ (1 mark)

0°, 120° B1

The probability that it rains on a certain day is 0.8. If it rains the probability that Onguti comes to school late is 0.7 but otherwise it is 0.4. If he comes to school late, the probability that he fails an exercise is 0.8 but if he comes early, the probability of failing an exercise is 0.3

(a) Draw a tree diagram to represent this information (2 marks)



B1 ✓ tree

B1 ✓ probabilities

(b) Determine

(i) The probability that it rains, he comes to schools early and he fails the exercise (2 marks)

$0.8 \times 0.3 \times 0.3$ M1

$= 0.072$. A1

(ii) The probability that he passes his exercise (2 marks)

$(0.8 \times 0.7 \times 0.2) + (0.8 \times 0.3 \times 0.7) + (0.2 \times 0.4 \times 0.2) + (0.2 \times 0.6 \times 0.7)$ M1

$= 0.38$ A1.

(iii) The probability that he comes to school late (2 marks)

$(0.8 \times 0.7) + (0.2 \times 0.4)$ M1

$= 0.64$. A1.

(iv) The probability that he comes to schools late and he passes his exercise (2 marks)

$(0.8 \times 0.7 \times 0.2) + (0.2 \times 0.4 \times 0.2)$ M1

$= 0.128$ A1.

The points A (1,4), B(-2,0) and C(4,-2) of a triangle are mapped onto $A^1(7,4)$ $B^1(x,y)$ and $C^1(10,16)$ by a transformation $N = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

(i) Determine the matrix N of the transformation (4 marks)

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & -2 & 4 \\ 4 & 0 & -2 \end{pmatrix} = \begin{pmatrix} 7 & x & 10 \\ 4 & y & 16 \end{pmatrix}$$

$a+4b=7$
$-2a+0=x$
$4a-2b=10$ M_1

$4a-2b=10$
$4a+16b=28$
$18b=18$
$b=1$

$4(a)-2(1)=10$
$4a=12$
$a=3$ M_1

$c+4d=4$
$8c-4d=32$
$9c=36$
$c=4$
$4+4d=4$
$4d=0$
$d=0$ A_1 all ✓

$M=\begin{bmatrix} 3 & 1 \\ 4 & 0 \end{bmatrix}$ B_1

(ii) Co-ordinates of B^1 (2 marks)

$-2a=x$
$x=-2(3)=-6$ M_1
$-2c=y$
$y=-2(4)=-8$

$B'(x,y) \rightleftharpoons B(-6,-8)$. A_1

(iii) $A^{II}B^{II}C^{II}$ are the image of $A^1B^1C^1$ under transformation represented by matrix $M=\begin{pmatrix} 2 & -1 \\ 0 & 0 \end{pmatrix}$
Write down the co-ordinates of $A^{II}B^{II}C^{II}$ (2 marks)

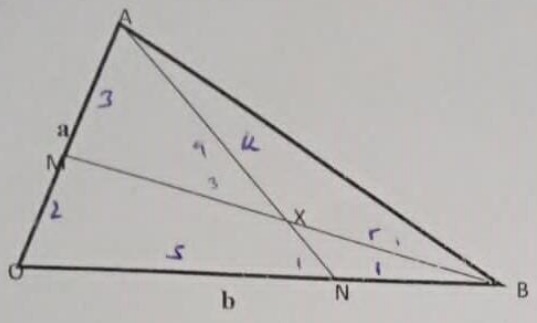
$$A''B''C'' = \begin{pmatrix} 2 & -1 \\ 0 & 0 \end{pmatrix}\begin{bmatrix} 7 & -6 & 10 \\ 4 & -8 & 16 \end{bmatrix} = \begin{bmatrix} 10 & -4 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$ M_1

$A''(10,0)$
$B''(-4,0)$
$C''(4,0)$ A_1

(vi) A transformation N followed by M can be represented by a single transformation K. Determine K (2 marks)

$$K = MN = \begin{pmatrix} 2 & -1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 4 & 0 \end{pmatrix} = \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}$$ M_1 A_1

In triangle OAB below **OA = a**, **OB = b** point M lies on **ON** such that **OM: MA**= 2:3 and point N lies on **OB** such that **ON** = $\frac{5}{6}$**OB** line AN intersect line MB at X.



(a) Express in terms of **a** and **b**

i) AN $= -\underset{\sim}{a} + \frac{5}{6}\underset{\sim}{b}$. B1 (1 mark)

ii) BM $= -\underset{\sim}{b} + \frac{2}{5}\underset{\sim}{a}$. B1 (1 mark)

b). Given that **AX**=k**AN** and **BX**=r**BM**,where where k and r are scalars.

i). write down two different expression for **OX** in terms of a,b,k and r. (2 marks)

$\underset{\sim}{OX} = \underset{\sim}{a} + k\left(-\underset{\sim}{a} + \frac{5}{6}\underset{\sim}{b}\right)$ $\underset{\sim}{OX} = \underset{\sim}{b} + r\left(-\underset{\sim}{b} + \frac{2}{5}\underset{\sim}{a}\right)$

$= (1-k)\underset{\sim}{a} + \frac{5}{6}k\underset{\sim}{b}$ B1 $= (1-r)\underset{\sim}{b} + \frac{2r}{5}\underset{\sim}{a}$. B1

ii). Find the value of k and r. (5 marks)

$1 - k = \frac{2r}{5}$ — i
$\frac{5}{6}k = 1 - r$ — ii } M1

$5 - 5k = 2r \Rightarrow 2r + 5k = 5$
$5k = 6 - 6r \Rightarrow 6r + 5k = 6$ M1
$4r = 1$
$r = 1/4$. A1

$k = 1 - \frac{2}{5}\left(\frac{1}{4}\right)$ M1
$= 1 - \frac{1}{10}$
$= \frac{9}{10}$. A1

iii). Determine the ratio in which x divides line MB. (1 marks)

$MX : XB = 3 : 1$ B1

URANGA MATHEMATICS ASSOCIATION 2023
TERM ONE FORM FOUR PP2 MARKING SCHEME.

the table for the function $y = x^3 + 4x^2 - x - 3$ for the domain $-5 \le x \le 2$ (2 marks)

X	-5	-4	-3	-2	-1	0	1	2
y	-23	1	9	7	1	-3	1	19

B_1 3-4✓ / B_2 all ✓

(b) On the grid provided, draw the graph of $y = x^3 + 4x^2 - x - 3$ for the domain $-5 \le x \le 2$
Use scale 1cm represent 1 unit horizontally 1cm represent 5 units vertically (3 marks)

(c)Use your graph to solve the following equations

i) $x^3 + 4x^2 - x - 3 = 0$. (2 marks)

$x_1 = -3.9$ $x_3 = 0.9$. B_1 2✓ or

$x_2 = -0.9$ B_2 3✓.

ii) $2x^3 + 8x^2 - 14x - 20 = 0$ (3 marks)

$y = x^3 + 4x^2 - x - 3$.

$0 = x^3 + 4x^2 - 7x - 10$.

$y = 6x + 7$ B_1

x	1	0	-1
y	13	7	1

$x_1 = -2.3$.
$x_2 = -1$
$x_3 = 2$. A_1