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School: .....M/S..... Candidate's Signature .....

121/2
Mathematics Alt. A
FORM FOUR.
OCTOBER 2022.
2 ½ Hours.

URANGA MATHEMATICS ASSOCIATION-2022.

Kenya Certificate of Secondary Education

MATHEMATICS
121/2
FORM FOUR
TIME: 2 ½ HOURS

INSTRUCTIONS TO CANDIDATES:

- Write your name, school, admission number and sign in the spaces provided above.
- This paper contains **TWO** sections: Section **I** and Section **II**.
- Answer **ALL** the questions in Section **I** and **FIVE** questions from section **II**.
- All answers and working **MUST** be written on the question paper in the spaces provided below each question.
- Marks may be given for correct working even if the answer is wrong.
- Non-programmable silent electronic calculators and KNEC Mathematical tables may be used, except where stated otherwise.

FOR EXAMINERS USE ONLY

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

SECTION II

17	18	19	20	21	22	23	24	Total

Grand Total

This paper consists of 15 printed pages. Candidates should check to ascertain that all pages are printed as indicated and that no questions are missing.

SECTION I (50 Marks)

Answer ALL the questions in this section.

1. Use logarithms to evaluate:

$$\sqrt[3]{\frac{45.3 \times 0.00697}{0.534}}$$

(3 marks)

No	S.F	log
45.3	4.53×10^{1}	1.6561
.00697	6.97×10^{-3}	$\bar{3}.8432^{+}$
		$\bar{1}.4993$
0.534	5.34×10^{-1}	$\bar{1}.7275^{-}$
		$\bar{1}.7718 \times \frac{1}{3}$

No	S.F	log
0.83933	8.3933×10^{-1}	$\bar{1}.9239$

$= 0.83933$

B_1 ✓ logs

M_1 ✓ divisio-

A_1

2. a) Expand $\left(1 - \frac{1}{2}x\right)^6$ to fourth term. (2 marks)

$1(1^6)\left(\frac{1}{2}x\right)^0 - 6(1^5)\left(\frac{1}{2}x\right)^1 + 15(1)^4\left(\frac{1}{2}x\right)^2 - 20(1)^3\left(\frac{1}{2}\right)^3 - M_1$

$= 1 - 3x + \frac{15}{4}x^2 - \frac{5}{2}x^3 \longrightarrow A_1$

b) Use the expansion above to evaluate $(0.98)^6$ (2 marks)

$0.98 = 1 - \frac{1}{2}x$

$x = 0.04.$

$1 - 3(0.04) + \frac{15}{4}(0.04)^2 - \frac{5}{2}(0.04)^3$ ——— M_1

$= 0.88584$ ——— A_1.

3. The price of a new car is shs. 800,000. If it depreciates at a constant rate to shs. 550,000 within 4 years, find the annual rate of depreciation. (3 marks)

$550{,}000 = 800{,}000\left(1 - \frac{r}{100}\right)^4$ M_1

$0.6875 = \left(1 - \frac{r}{100}\right)^4$

$0.91058 = 1 - \frac{r}{100}$ m_1

$\frac{r}{100} = 0.08941985($

$r = 8.941985658$

accept 8.942%

4. Object A of the area $10cm^2$ is mapped onto its image B of area $60cm^2$ by a transforma whose matrix is given by $P = \begin{pmatrix} x & 4 \\ 3 & x+3 \end{pmatrix}$. Find the positive values of x. (3 marks)

$x(x+3) - 4(3) = \frac{60}{10}$ M_1

$x^2 + 3x - 12 = 6$

$x^2 + 3x - 18 = 0$.

$(x+6)(x-3) = 0$. M_1

$x = -6$ ignore

$x = 3$ ——— A_1.

5. Without using a calculator or mathematical tables, express $\frac{\sqrt{3}}{1-Cos30^0}$ in surd form simplify. (3 marks)

$\frac{\sqrt{3}}{1-\frac{\sqrt{3}}{2}}$ B_1

$= \frac{2\sqrt{3}}{2-\sqrt{3}}$

$\frac{2\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$ M_1

$= \frac{4\sqrt{3}+6}{4-3}$

$= \frac{6+4\sqrt{3}}{1} = 6+4\sqrt{3}$. A_1.

6. Pipe A can fill a tank in 2 hours, Pipe B and C can empty the tank in 5 hours and 6 ho respectively. How long would it take:

a) To fill the tank if A and B are left open and C is closed. (2 marks)

A & B in 1hr.

$= \frac{1}{2} - \frac{1}{5} = \frac{3}{10}$ of tank

$\Rightarrow 1 \times 1 \times 10/3 = 3hrs\ 20m$ M_1 A_1.

b) To fill the tank with all pipes open. (2 marks)

A B & C

$= \frac{3}{10} - \frac{1}{6} = \frac{2}{15}$

$\Rightarrow 1 \times 1 \times \frac{15}{2} = 7\frac{1}{2}\ hrs$ M_1 A_1.

7. The position vector of A and B are $a = 4\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}$ and $b = 10\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}$. D is a point on AB such that AD:DB is 2:1. Find the co-ordinates of D. (2 marks)

$OD = \frac{1}{3}\begin{pmatrix}4\\4\\-6\end{pmatrix} + \frac{2}{3}\begin{pmatrix}10\\4\\12\end{pmatrix}$ —— M1

$= \begin{pmatrix}4/3\\4/3\\-2\end{pmatrix} + \begin{pmatrix}20/3\\8/3\\24/3\end{pmatrix} = \begin{pmatrix}8\\4\\6\end{pmatrix}$

$D(8, 4, 6)$ —— A1.

8. Given that y is inversely proportional to x^n and k is the constant of proportionality and that x=2, when y=4.5 , and x=3, when y=4/3. Find the values of n and k. (4marks)

$y = \frac{k}{x^n}$ —— M1

$4.5 = \frac{k}{2^n}$

$k = 4.5(2^n)$.

$\frac{4}{3} = \frac{k}{3^n}$

$k = \frac{4}{3}(3^n)$

$4.5(2^n) = \frac{3}{2}\frac{4}{3}(3^n)$ —— M1

$\frac{4.5 \times 3}{4} = \left(\frac{3}{2}\right)^n$

$3.375 = 1.5^n$

$n = \frac{\log 3.375}{\log 1.5} = 3$.

$k = (4.5)(2)^3$ —— M1

$= 36$ —— A1.

9. The coordinates of the end points of diameter are A(2,4) B(–2,6). Find the equation of a circle in the form $ax^2 + by^2 + cx + dy + e = 0$ (3 marks)

Centre $\left(\frac{2+-2}{2}, \frac{4+6}{2}\right)$

$= (0, 5)$

Dia $= \sqrt{4^2 + (-2)^2} = \sqrt{20}$. } B1 Both ✓

radius $= \frac{1}{2}\sqrt{20} = \sqrt{5}$.

$(x-0)^2 + (y-5)^2 = 5$ —— M1

$x^2 + y^2 - 10y + 20 = 0$ —— A1

20

10. Given that the mean of 9, 8, 5, 5 and 8 is 7; find the standard deviation of the numbers to 2 d.p (3 marks)

$n = 5$

x	9	8	5	5	8
d	2	1	-2	-2	1
d^2	4	1	4	4	1

B_1

Std. $\sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{14}{5}}$ —— M_1

$= 1.67332$

~~1.67~~ ✓ A_1 CAO

11. A vendor mixed grade A rice and grade B rice in the ratio 1:3 to form a mixture which she sold at sh.105 making a profit of 40%. Given that the cost price of grade B rice is sh.80 per kg. Find the cost price of 1kg grade A rice. (3marks)

$140\% = 105$

$100\% =$

$\frac{100}{100} \times 105 \times \frac{100}{140}$

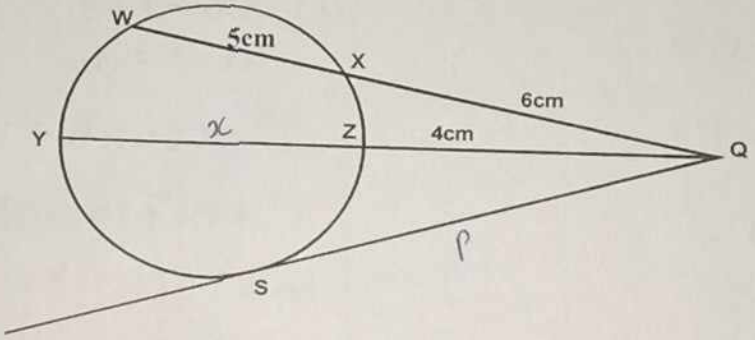
$= 75/=$ B_1

$\frac{1}{4}A + \frac{3}{4}(80) = 75$ —— M_1

$A = 15 \times 4$

$= 60/=$ —— A_1.

12. Chord WX and YZ intersect externally at Q. The secant WQ =11cm and QX =6cm while ZQ=4cm



a) Calculate the length of chord YZ (2 marks)

$11 \times 6 = (4+x)4$ M_1

$\frac{66}{4} = 4 + x$

$x = \frac{66}{4} - 4$

$= 12.5\text{cm}$ A_1.

b) Find the length of the tangent SQ (2 marks)

$P^2 = 16.5 \times 4$ — M_1

$P = \sqrt{66} = 8.124$ A_1.

13. Given that x, y and z are integers and that $8 \leq x \leq 10,\ 5 \leq y \leq 7, 4 \leq z \leq 6$.

Find the percentage error in $\frac{x+y}{z}$ (3 marks)

Max quotient $= \frac{10+7}{4} = \frac{17}{4}$

True quotient $= \frac{9+6}{5} = 3$

Min quotient $= \frac{8+5}{6} = \frac{13}{6}$

$Ae = \left(\frac{17}{4} - \frac{13}{6}\right) \times \frac{1}{2} = \frac{25}{24}$ M1

$\%e = \left(\frac{25}{24} \div 3\right) \times 100$ – M1

$\frac{25}{72} \times 100$

$= 34.7222\%$

accept 34.72% } M1

14. Solve the equation below by completing the square. $5 - 9x - 2x^2 = 0$ (2 marks)

$-2x^2 - 9x + 5 = 0$

$x^2 - \frac{9}{-2}x + \frac{5}{-2} = 0$

$x^2 + \frac{9}{2}x = \frac{5}{2}$

$x^2 + \frac{9}{2}x + \frac{81}{16} = \frac{5}{2} + \frac{81}{16}$ M1

$\left(x + \frac{9}{4}\right)^2 = \frac{121}{16}$

$x + \frac{9}{4} = \pm \frac{11}{4}$

$x = \pm \frac{11}{4} - \frac{9}{4}$

$x = 1/2$

$x = -5$ } A1

15. Solve the simultaneous equations

$2x - y = 3$

$x^2 - xy = -4$ (3marks)

$2x - y = 3$

$2x - 3 = y$

$x^2 - x(2x-3) = -4$

$x^2 - 2x^2 - 3x = -4$

$-x^2 - 3x + 4 = 0$ M1

$x = \frac{-3 \pm \sqrt{9+16}}{-2}$

$x = \frac{-3+5}{-2}$

$= -1,$ y

$=$

16. Solve the equation

$\log_2(2+3x) + 3\log_2 2 = 2 + \log_2(2x+6)$ (3marks)

$\log_2(2+3x) + 3\log_2 2 = 2\log_2 2 + \log_2(2x+6)$

$\log_2\{(2x+3)(8)\} = \log_2(4(2x+6))$ ——— M1

$\frac{8(2x+3)}{4} = \frac{4(2x+6)}{4}$

$2(2x+3) = (2x+6)$

$4 + 6x = 2x + 6$ ——— M1

$4x = 2$

$x = 1/2$ ——— A1.

SECTION II(50 Marks)

Answer Any Five Questions In This Section

17. Two fair dice one a regular tetrahedron (4 faces) and the other a cube are thrown. The scores are added together.

a) Draw a table to show all possible outcomes. (2 marks)

Tetrahedron \ Die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10

B_2

b) Find the probability that

i) The sum is 6. (1 mark)

$P(6) = \frac{4}{24} = \frac{1}{6}$ B_1

ii) The sum is an odd number. (1 mark)

$P(\text{odd}) = \frac{12}{24} = 1/2$ B_1

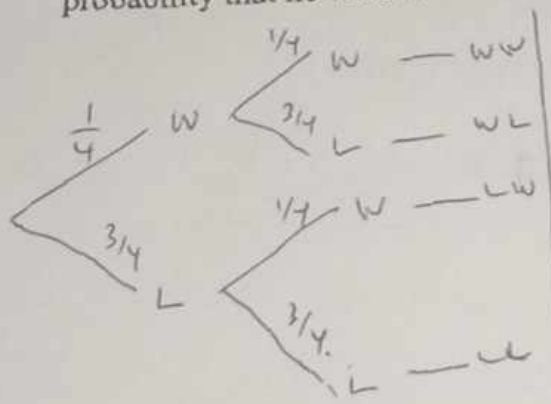
iii) The sum is 6 or 9. (2 marks)

$P(6 \text{ or } 9) = P(6) + P(9)$ → M_1

$= \frac{1}{6} + \frac{2}{24}$ → A_1

$= 1/4$

c) If a player wins a game by throwing a sum of 6 or 9, draw a tree diagram and use it to find probability that he wins at least once when the dice are thrown twice. (4 marks)



B_2 ✓two

$1 - (3/4 \times 3/4)$ —— M_1

$1 - 9/16$

$= \frac{7}{16}$ —— A_1

$P(\text{atleast Once}) = 1 - P(LL)$

18. The Hire Purchase (H.P) price of a public address system was Ksh 448 000. A deposit of Ksh 112 000 was paid followed by 24 equal monthly instalments. The cash price of the public address system was 15% less than the H.P price.

a) Calculate :

(i) The monthly instalment. (2 marks)

$I = \frac{448000 - 112000}{24}$ M1

$= 14,000$ /= A1

(ii) The cash price. (2marks)

$CP = \frac{85}{100} \times 448000$ ——— M1

$= 380,800$ /= ——— A1

b) A customer decided to buy the system in cash and was allowed an 8% discount on the cash price. He took a bank loan to buy the system in cash. The bank charged compound interest on the loan at rate of 16% p.a. compounded quarterly. The loan was repaid in 2years. Calculate the amount repaid to the bank by the end of the 2 years. (3 marks)

$\frac{92}{100} \times 380,500$ → M1

$= 350,336$

$A = 350,336\left(1 + \frac{16}{400}\right)^4$ → M1

$= 409,843.57$ accept 409844 → A1.

c) Express as a percentage of the Hire Purchase price, the difference between the amounts repaid to the bank and the Hire Purchase price. (3 marks)

$448000 - 409844$ → M1

$= 38,156$

$\frac{38156}{448000} \times 100$ → M1

$= 8.5170606\ \%$ → A1.

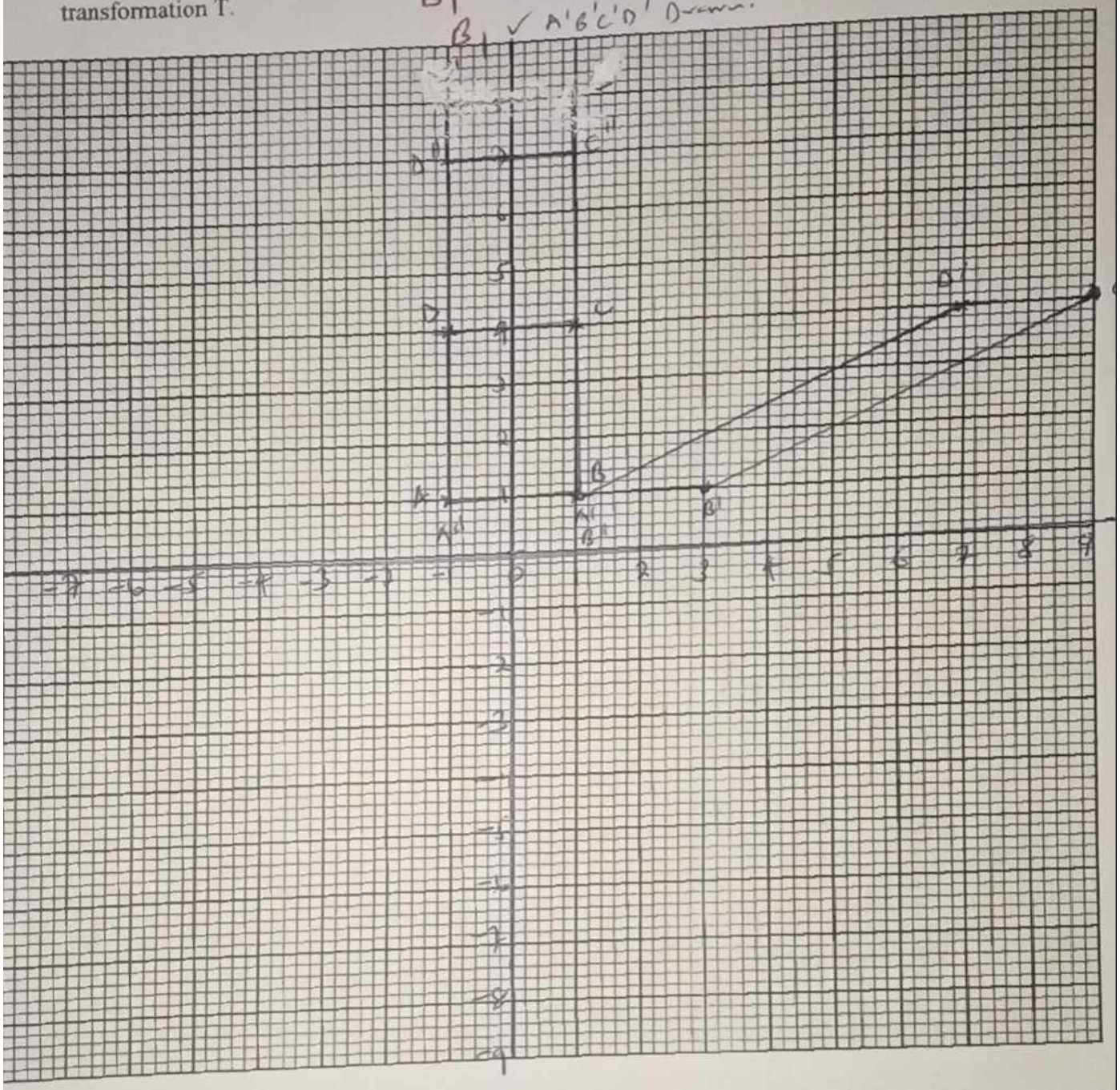
accept $8.517\ \%$

19. The vertices of a rectangle ABCD are (-1, 1) , (1, 1) ,(1, 4)and (-1, 4)respectively. The vertices of its image under transformation T areA' (1,1), B'(3,1) C' (9, 4) and D'(7, 4)

a)(i)Draw on the grid provided rectangle ABCD and its image A'B'C'D' under the transformation T. (2 marks)

B1 ✓ABCD Drawn

B1 ✓ A'B'C'D' Drawn.



(ii) Describe fully the transformation. (2 marks)

Shear wrt X-axis Invariant B1 // parallel to X-axis

wrt C(1, 4) ⟹ C'(9, 4) or scale factor 2. B1

(iii) Determine the matrix of transformation. (3 marks)

$\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$-1 + k = 1$

$k = 2$

$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

M_1 ✓attempt to find k

A_1 ✓k value

B_1 ✓matrix.

(iv) On the grid as in (a), draw rectangle A"B"C"D", the image of rectangle ABCD under a stretch with line y=1 invariant and stretch factor 2. State the coordinates ofA"B"C"D" (3 marks)

$A''(-1,1)$ $B''(1,1)$ $C''(1,7)$ $D''(-1,7)$ B_1 ✓coordinates

B_1 ✓A"B"C"D" drawn.

20. An arithmetic progression of 41 terms is such that the sum of the first five terms is 560 and the sum of the last five terms is -250. Find:

a) The first term and the common difference (6 marks)

$S_5 = \frac{5}{2}(2a + 4d) = 560$

$S_n - S_{36} \Rightarrow$

$\frac{41}{2}(2a + 40d) - \frac{36}{2}(2a + 35d) = -250$ — M_1 Both expression ✓

$5a + 190d = -250$

$5a + 10d = 560$ — M_1 Both equations ✓

$180d = -810$ — M_1 ✓ attempt to solve either a or d

$d = -4.5$ — A_1 ✓ value

$a = \frac{560 + 45}{5}$ — M_1

$= 121$ — A_1

(2marks)

b) The last term

$T_{41} = a + 40d$.

$121 + 40(-4.5)$ — M_1

-59 — A_1.

(3 marks)

c) The sum of the progression

$S_{41} = \frac{41}{2}(242 + 40(-4.5))$ — M_1

$= 1271$ — A_1.

02

$S_{41} = \frac{41}{2}(121 + (-59))$.

$= 1271$.

21. Three quantities x, y and z are such that x varies directly as the square of y and inversely as the square root of z.

(a) (i). Given that $x = 12$, and $y = 24$, and $z = 36$, find x when $y = 27$ and $z = 121$ (3 marks)

$x = \frac{ky^2}{\sqrt{z}}$

$12 = \frac{576k}{6}$ M_1

$k = 0.125$

$x = \frac{0.125 \times 27^2}{\sqrt{121}}$ ——— M_1

$= 8\frac{25}{28}$ accept 8.284 A_1

(ii). If y increases by 5% and z decreases by 19%, find the percentage increase in x (4 marks)

$x' = \frac{k(1.05y)^2}{\sqrt{0.81z}}$ M_1

$= \frac{1.225y^2}{\sqrt{z}}$ M_1

$\left(\frac{1.225}{1} - 1\right) \times 100$ M_1

22.5 % A_1

(b) If y is inversely proportional to the square root of x and that $x = 4$ when $y = 3$, calculate the value of x when $y = 8$. (3 marks)

$y = \frac{k}{\sqrt{x}}$

$3 = \frac{k}{2}$ ——— M_1

$k = 6$.

$8 = \frac{6}{\sqrt{x}}$ M_1

$\sqrt{x} = 3/4$.

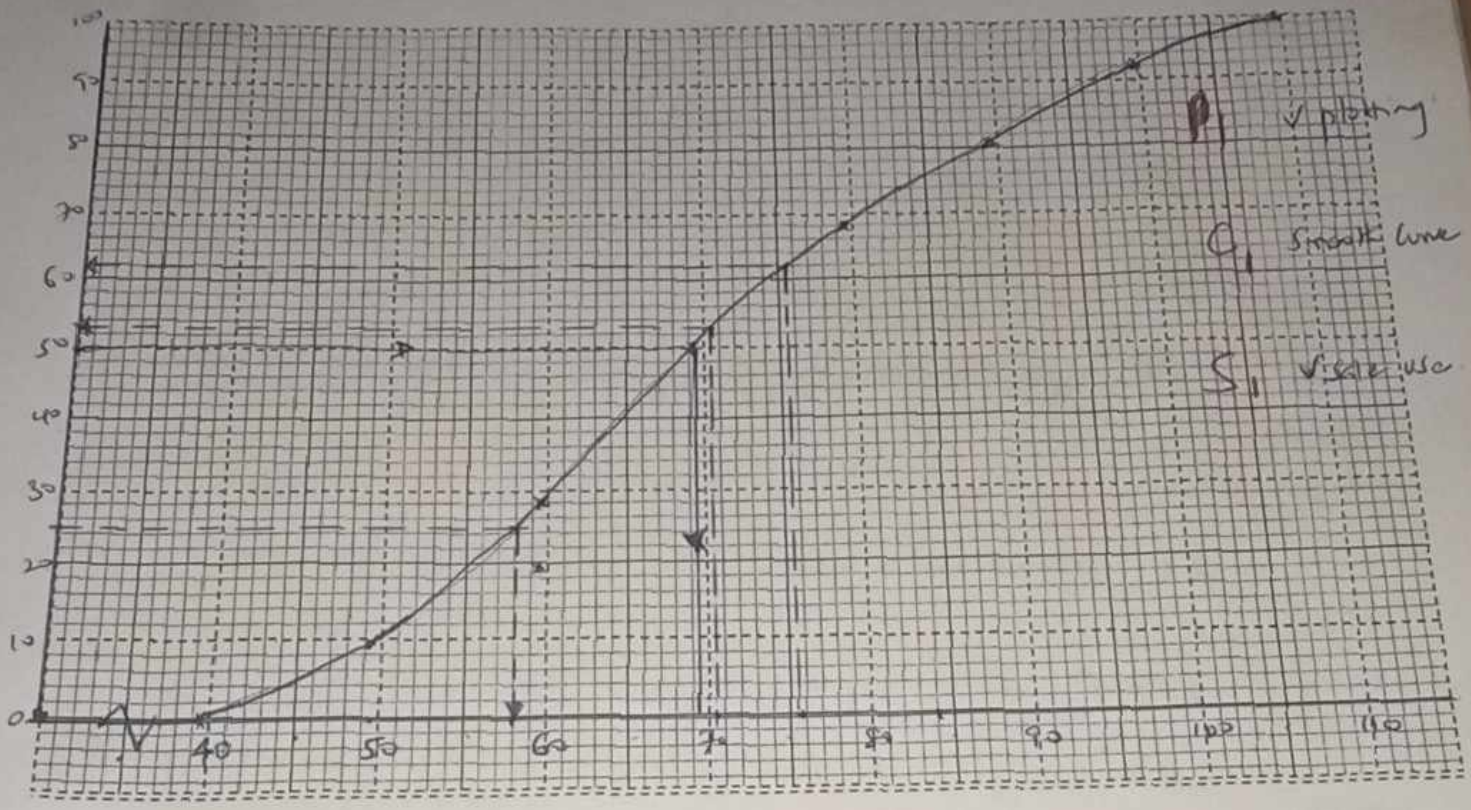
$x = 9/16$ accept 0.5625 A_1.

22. The table below shows the masses measured to the nearest Kg of 100 people.

Mass kg	40-49	50-59	60-69	70-79	80-89	90-99	100-109
No of people	9	19	22	18	13	11	8
cf	9	28	50	68	81	92	100

B1

a) Draw a cumulative frequency curve for the data above. (4 marks)



b. Use your graph to estimate

i. The median mass. (1 mark)

69.5 kg B1

ii. The number of people whose mass lies between 70.5 kg and 75.5 kg (2 mark)

53 − 62 ⇒ 10 people. ± 1

B1

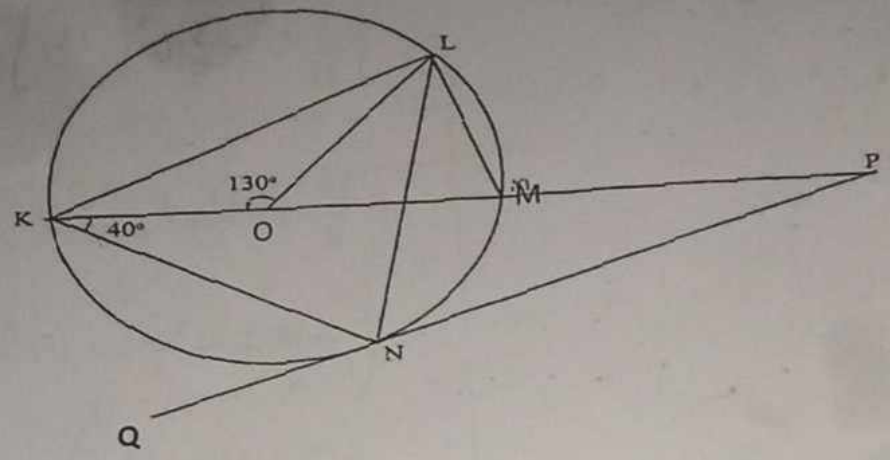
c. From your graph find the interquartile range B1 (3 marks)

$Q_1 = 58$ } B1
$Q_3 = 84$ }

$84 - 58$ ——— M1

$= 26$ ——— A1.

23. In the figure below, K L M and N are points on the circumference of a circle centre O. The points K, O, M and P are on a straight line. PQ is a tangent to the circle at N. Angle KOL = 130^0 and angle MKN = 40^0



Find the values of the following angles, stating the reasons in each case:

a. <MLN (2mks)

40° Angles subtended by a common chord to the circumference (in the same segment) are equall. B1 B1

b. <OLN (2mks)

25° Diameter subtends a right angle at the circumference B1

B1 Accept any other true reason

c. < LNP (2mks)

65° Angles in the alternate segments are equal. B1

B1

d. <MPQ (2mks)

10°; Angle sum of a triangle is supplementary B1

B1

e. <KNQ (2mks)

50° Angles in a straight line add upto 180°. B1

B1 (accept other true reason)

24. A particle moves in straight line so that t seconds after passing a fixed point in the line, its velocity V m/s is given by $V = \frac{1}{2}t^2 - 3t + 7$. Calculate;

a) The velocity after 8 seconds (2 marks)

$V = \frac{1}{2}t^2 - 3t + 7$

$V = \frac{1}{2}(8^2) - 3(8) + 7$ ——— M_1

$= 15 \text{ m/s}$ ——— A_1

b) The acceleration when t = 0 (2 marks)

$a = \frac{dv}{dt} = t - 3$

at $t = 0$

$a = 0 - 3 = -3 \text{m/s}^2$ M_1 A_1

c) The minimum velocity (2 marks)

min vel ($a = 0$)

$t - 3 = 0$

$t = 3$ sec.

$v = \frac{1}{2}(3^2) - 3(3) + 7$ ——— M_1

$v = 4.5 - 2.5 \text{ m/s}$ ——— A_1

d) The distance traveled in the 1st two seconds of motion. (2 marks)

$S = \int v\,dt$

$S = \int_0^2 (\frac{1}{2}t^2 - 3t + 7)\,dt$

$= \frac{1}{6}t^3 - \frac{3}{2}t^2 + 7t \Big]_0^2$ M_1

$= (\frac{1}{6}(8) - \frac{3(4)}{2} + 7(2)) - (0)$

$= 9\frac{1}{3} \text{ m}$ A_1.

e) The distance traveled in the third second. (2 marks)

$\int_2^3 (\frac{1}{2}t^2 - 3t + 7)\,dt$

$\left[\frac{1}{6}t^3 - \frac{3}{2}t^2 + 7t\right]_2^3$ M_1

$= (27/6 - \frac{27}{2} + 21) - (9\frac{1}{3})$

$= (12 - 9\frac{1}{3})$

$= 2\frac{2}{3} \text{ m}$ A_1.