URANGA MATHEMATICS ASSOCIATION – 2021

FORM FOUR TERM THREE PP1

MARKING SCHEME.

SECTION I (50 Marks)

Answer ALL the questions in the spaces provided.

1. Evaluate without using mathematical tables or calculators (3 marks)

$$\sqrt{\frac{0.0625\times 2.56\times 0.8}{0.25\times 0.08\times 0.5\times 0.2}\times 10000000\times 10000000}$$ – M1

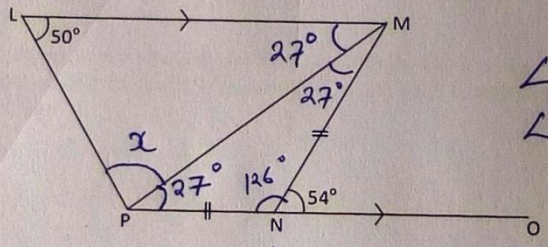
$$= \sqrt{\frac{625\times 256\times 8}{25\times 8\times 5\times 2\times 10}}$$ – M1

$= \sqrt{64}$

$= 8$. – A1

03

2. In the figure below ∠MNO = 54° and ∠PLM = 50°, PN = NM and PO is parallel to LM. Find the value of ∠LPM. (3 marks)



∠PNM = 126° – B1

∠PML = 27° – B1

$x = 180° - (50° + 27°)$

$x = 103°$. – B1

03

3. Solve for x and y in the pair of equations below: (3 marks)

$3^{(2x-y)} = 27$

$4^x \div 16^y = 1$

$3^{(2x-y)} = 3^3$

$4^x \div 4^{2y} = 4^0$ } M1

$(2x - y = 3)1$

$(x - 2y = 0)2$

$2x - y = 3$ – M1

$2x - 4y = 0$ –

$3y = 3$

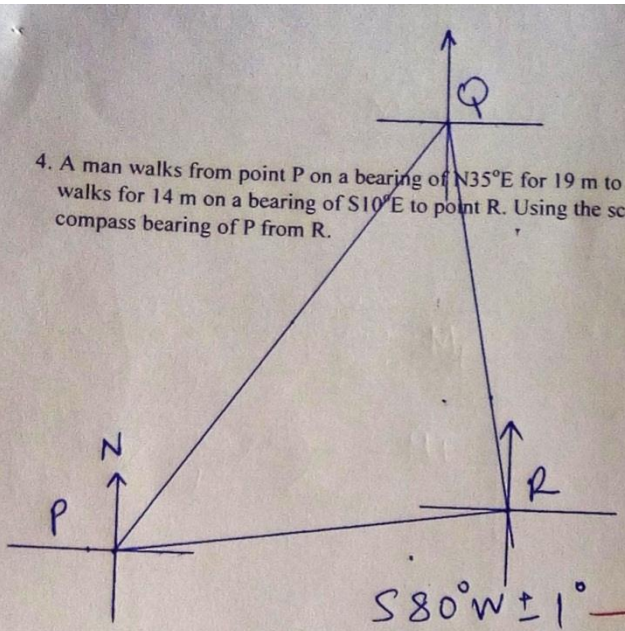
$y = 1$.

$x = 2$. } – A1

03.

4. A man walks from point P on a bearing of N35°E for 19 m to point Q. At Q, he alters course and walks for 14 m on a bearing of S10°E to point R. Using the scale of 1:200, determine the compass bearing of P from R.

(3 marks)



S_1 – Scale

B_1 – Complete diagram.

S80°W ± 1° — B_1

03

5. A trader bought 10 shirts and 8 blouses for a total of Ksh 29 000. She sold each shirt at 18% profit and each blouse at 20% profit. If she realized a total sales of Ksh 34 460, what was the cost price of each item?

(4 marks)

Let a shirt cost s and blouse b.

$(10s + 8b = 29{,}000)\,11.8$
$(11.8s + 9.6b = 34{,}460)\,10$ } — M_1 – Both equations

$118s + 94.4b = 342{,}200$
$-\ 118s + 96b = 344{,}600$ } M_1

$-1.6b = -2400$

$b =$ sh. 1500. — A_1

$10s + 8(1500) = 29000$

$s =$ sh. 1700. — B_1

04.

6. Simplify completely $(3x+2y)^2 - (3x-2y)^2$.

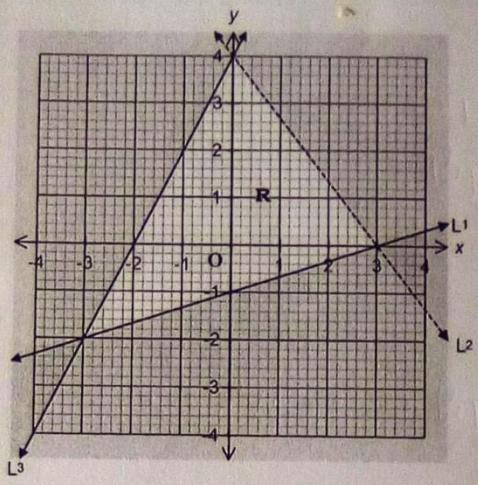
(2 marks)

$(9x^2 + 12xy + 4y^2) - (9x^2 - 12xy + 4y^2)$ — M_1

$= 24xy$ — A_1

02.

7. Form the three inequalities that satisfy the given region R. (3 marks)



L_1

$(0,-1)(3,0)$

$m = \frac{0+1}{3-0} = \frac{1}{3}$.

$y = \frac{1}{3}x - 1$.

$y \geq \frac{1}{3}x - 1$. $-B_1$

L_2

$(0,4)(3,0)$

$m = \frac{0-4}{3-0} = -\frac{4}{3}$.

$y = -\frac{4}{3}x + 4$.

$y < -\frac{4}{3}x + 4$. $-B_1$

L_3

$(-2,0)(0,4)$

$m = \frac{4-0}{0+2} = 2$.

$y = 2x + 4$.

$y \leq 2x + 4$. $-B_1$ 03

N/B: Check alternative forms.

8. Makau, Wanjiru and Kemboi start a race at 9.03 a.m in the same direction to run round a circular course. Makau makes the circuit in 252 seconds, Wanjiru in 308 seconds and Kemboi in 198 seconds. If they start from the same point, at what time will they next be all at the starting point together (3 marks)

2	252	308	198
2	126	154	99
3	63	77	99
3	21	77	33
7	7	77	11
11	1	11	11
		1	1

$Lcm = 2^2 \times 3^2 \times 7 \times 11$

$= 2772$ sec

$= \frac{2772}{60}$ } M1

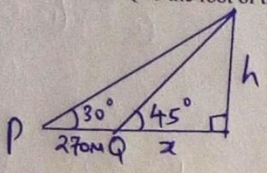
$= 46$ minutes 12 sec

$= 9.03$
$\quad 46.12$ } M1

$9.49.12$ Am. A1

03

9. From a point P, a man notices that the angle of elevation of the top of a building is 30°. He moves 270 metres towards the foot of the building from P to Q and notices that the new angle of elevation is 45°. Given that P and Q are on the same side of the building as P, calculate the distance from Q to the foot of the building. (3 marks)



$\tan 45° = \frac{h}{x}$

$h = x \tan 45°$ ✓

$\tan 30° = \frac{h}{x+270}$

$h = (x+270)\tan 30°$ ✓ } M1

$x \tan 45 = (x+270)\tan 30°$

$x = (x+270)0.5774$ – M1

$0.4226x = 15.5898$

$x = 36.89m.$ – A1

03.

10. Using logarithm tables evaluate to 4 significant figures; (4 marks)

$$\frac{\sqrt{154 \times 0.07884}}{(12.5)^2}$$

No	Log.
154	2.1875
0.07884	$\bar{2}.8967$
	$1.0842 \times \frac{1}{2}$
	0.5421
12.5	1.0969
	$\times 2$
	2.1938
	$\bar{2}.3483$

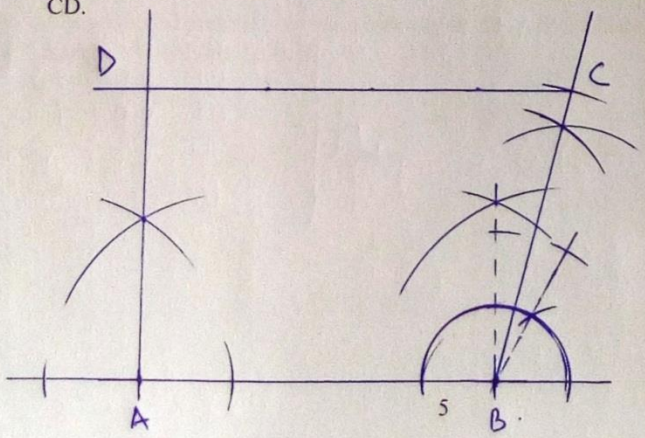
2.23×10^{-2} ← $\bar{2}.3483$

$= 0.02230.$

– M1 – All Logs
– M1 – Add / Sub
– M1 – Div / Multiplication
– B1 – Accuracy.

04.

11. Using a ruler and a pair of compasses only, construct a quadrilateral ABCD in which AB = 6cm, BC = 5cm, $\angle ABC = 105°$, $\angle DAB$ is a right angle and AB is parallel to CD. Measure CD. (3 marks)



B1 – 90° and 105° constructed
B1 – Complete diagram

CD = 7.2 ± 0.1cm. B1

03

12. Given the column vectors $\underset{\sim}{a} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}, b = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$ *and* $c = \begin{pmatrix} 5 \\ -10 \end{pmatrix}$ and $\mathbf{p} = 2\mathbf{a} - \frac{1}{2}\mathbf{b} + \frac{2}{5}\mathbf{c}$, express $\mathbf{p}$ as a column and hence calculate its magnitude $|\mathbf{p}|$ correct to 2 decimal places. (3 marks)

$\underset{\sim}{P} = 2\begin{pmatrix} -3 \\ 2 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 4 \\ -6 \end{pmatrix} + \frac{2}{5}\begin{pmatrix} 5 \\ -10 \end{pmatrix}$ — M_1

$\underset{\sim}{P} = \begin{pmatrix} -6 \\ 3 \end{pmatrix}$ —— A_1

$|\underset{\sim}{P}| = \sqrt{(-6)^2 + (3)^2}$

$= 6.71$ units. — B1

03.

13. Tap A fills a water tank in 30min, B in 20mins and C in 10mins. All three taps are turned on for 4 minutes and then C is turned off. How long will it take for the tank to be filled after C has been closed? (3 marks)

All the taps in 4 minutes

$= 4\left(\frac{1}{30} + \frac{1}{20} + \frac{1}{10}\right)$ — M_1

$= \frac{11}{15}$.

Remaining $= 1 - \frac{11}{15} = \frac{4}{15}$.

A and B in one minute

$= \frac{1}{30} + \frac{1}{20} = \frac{1}{12}$

Time taken to fill

$= \frac{4}{15} \times \frac{12}{1}$ — M_1

$= 3\frac{1}{5}$ minutes. — A1

or

3 min 12 sec.

03.

14. A container of height 30*cm* has a capacity of 1.5 *liters*. What is the height of a similar container of volume 3.0 m^3. (3 marks)

VSF $= \frac{1.5}{3000} = \frac{1}{2000}$. — M_1

LSF $= \sqrt[3]{\frac{1}{2000}}$

$= 0.07937$

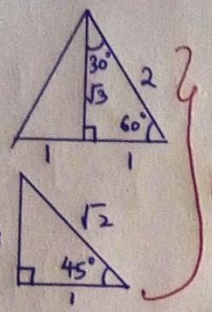
$h = \frac{30}{0.07937}$ — M_1

$h = 377.98$cm. — A1

03.

15. Without using calculator or mathematical tables, simplify (4marks)

$$\frac{\cos 30 - \sin 45}{\sin^2 30 + \tan^2 45}$$



$$= \frac{\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}}{\left(\frac{1}{2}\right)^2 + (1)^2} - M_1$$

$$= \frac{\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}}{1\frac{1}{4}}$$

$$= \left(\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\right)\frac{4}{5} - M_1$$

$$= \frac{2\sqrt{3}}{5} - \frac{4}{5\sqrt{2}}$$

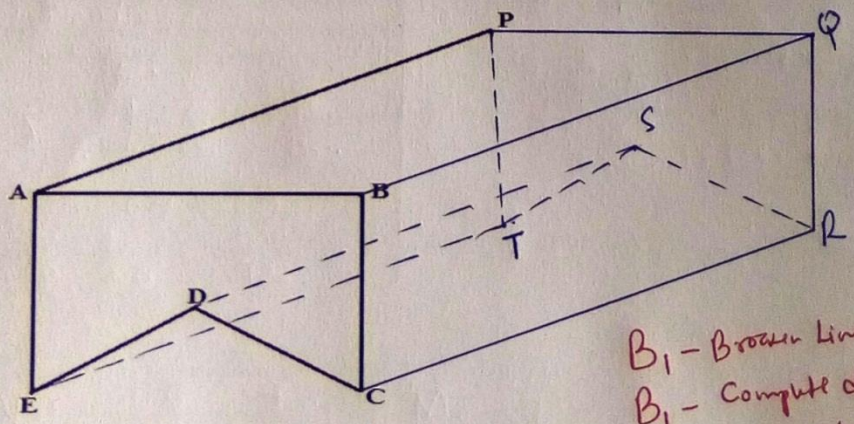
$$= \frac{2\sqrt{6} - 4.}{5\sqrt{2}}$$ A1

04

16. In the figure below ABCDE is a cross-section of a solid ABCDEPQRST. The solid has a uniform cross-section. Given that AP is an edge of the solid, complete the sketch showing the hidden edges with broken lines.

(3 marks)

1.



B_1 – Broken Lines.
B_1 – Complete diagram.
B_1 – Correct Labeling.

03.

SECTION II (50 MARKS)

Answer any FIVE questions in this section.

17. A salesman dealing in mattresses earns a basic salary and commission as follows:

	Commission
For sales up to Ksh 150 000	0%
For sales above Ksh 150 000	
First Ksh 85 000	3%
Next Ksh 85 000	4%
Any amount above Ksh 320 000	5%

a) In the month of November 2020, the salesman earned a basic salary of Ksh 35 000 and he sold 90 mattresses at Ksh 4 000 each. Calculate:

i) His total sales in the month of November 2020. (1 mark)

$= 90 \times 4000$

$= 360,000 =$ —— B1

ii) His total earnings that month. (3 marks)

$Comm = \frac{3}{100} \times 85,000 + \frac{4}{100} \times 85,000 + \frac{5}{100} \times 40,000$ — M1 $= Sh.\ 42,950$ A1

$= Sh.\ 7,950$

$Total\ Earning = 35,000 + 7950$ —— M1

b) In the month that followed, his basic salary was decreased by 10%. If he received a total earning of Ksh 36 970 in that month, calculate:

i) Total sales that month. (4 marks)

$BS = \frac{90}{100} \times 35,000$ —— M1

$= Sh.\ 31,500.$

$Comm = 36,970 - 31,500$ — M1

$= Sh.\ 5470$

$1^{st}\ Comm = \frac{3}{100} \times Sh.\ 85,000$

$= 2550 =$

$2^{nd}\ Comm = 5470 - 2550$

$= 2920$

$Sales = 150,000 + 85,000 + \frac{100}{4} \times 2920$ — M1

$= Sh.\ 308,000$ A1

ii) The number of mattresses sold in that month. (2 marks)

$= \frac{308,000}{4000}$ — M1

$= 77$ A1

10

18. a) A carpet measuring (x+4)m by (x-1)m laid down in a rectangular room measuring 2x m by x m leaving out uncovered floor near the walls round the room. If the carpet is $36m^2$, calculate the area of the uncarpeted floor. (6 marks)

$(x+4)(x-1) = 36$ — M1

$x^2 - x + 4x - 4 = 36$

$x^2 + 3x - 40 = 0$ — A1

$x = \frac{-3 \pm \sqrt{9 - 4(-40)}}{2}$ — M1

$x = \frac{-3 \pm 13}{2}$

$x = -8$ or 5

$x = 5M$ — A1

Area floor

$= 2(5) \times 5$ — M1

$= 50M^2$

Area of Uncarpeted floor

$= 50 - 36$

$= 14m^2$ — A1

b) If 20cm square tiles were to be used to carpet the uncarpeted section of the floor in (a) above, calculate the cost of carpeting the whole floor if the carpet costs sh.300 per square metre and each tile costs sh.100 ~~per square metre~~. (4 marks)

Area of one tile $= 20^2 = 400cm^2$

No of tiles $= \frac{140000}{400}$ — M1

$= 350$ tiles. — A1

Carpeting cost

$= 36 \times 300 =$

$= sh.10,800$

Tiling $= 350 \times 100$

$= sh.35,800$ } M1

Total

$= 10,800 + 35,000$

$= sh.45,800$ — A1

10

19. A sector of angle 108^0 is cut from a circle of radius 20 cm. It is folded to form a cone.
Calculate: (Take $\prod = 3.142$)

(a) The curved surface area of the cone. (2 marks)

$= \frac{108}{360} \times 3.142 \times 20^2$ — M1

$= 377.04 \text{ cm}^2$ — A1

(b) The base radius of the cone. (3 marks)

$L = \frac{108}{360} \times 2 \times 3.142 \times 20$ — M1

$= 37.704$

$2 \times 3.142 \times r = 37.704$ — M1

$6.284r = 37.704$

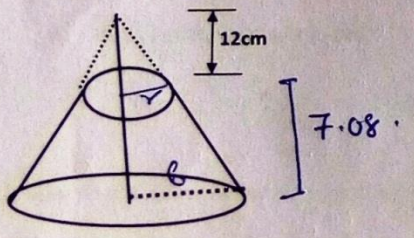
$r = 6 \text{ cm}.$ — A1

(c) The vertical height of the cone. (2 marks)

$h = \sqrt{20^2 - 6^2}$ — M1

$h = 19.08 \text{ cm}.$ — A1

(d) If 12 cm of the cone is chopped off to form a frustrum as shown below.



Calculate the volume of the frustum formed. (3 marks)

$\frac{6}{r} = \frac{19.08}{12.}$ — M1

$r = 3.774.$

$V = \left(\frac{1}{3} \times 3.142 \times 6^2 \times 19.08\right) - \left(\frac{1}{3} \times 3.142 \times 3.774^2 \times 12\right)$ — M1

$= 540.39 \text{ cm}^3.$ — A1

10

10

20. The velocity v m/s of a moving particle at time t seconds is given by:

$$v = 11t - 2t^2 - 5$$

a) Find:

i) The velocity of the particle when $t = 2$ seconds. (1 mark)

$V = 11(2) - 2(2)^2 - 5$

$= 9$ m/s. — B1

ii) The instants when the particle is momentarily at rest. (3 marks)

$-2t^2 + 11t - 5 = 0$. — M1

$t = \dfrac{-11 \pm \sqrt{11^2 - 4(-2 \times -5)}}{-4}$ — M1

$t = \dfrac{-11 \pm 9}{-4}$

$t = \frac{1}{2}$ sec or 5 sec. — A1

b) Find:

i) The time taken by the particle to attain maximum velocity. (2 marks)

$\dfrac{dv}{dt} = 11 - 4t = 0$ — M1

$-4t = -11$

$t = 2\frac{3}{4}$ sec. — A1

ii) The maximum velocity attained. (1 mark)

$V = 11\left(2\frac{3}{4}\right) - 2\left(2\frac{3}{4}\right)^2 - 5$

$= 10\frac{1}{8}$ m/s — B1

c) Calculate the distance travelled by the particle during the third second. (3 marks)

$\int_3^4 (11t - 2t^2 - 5)\,dt$

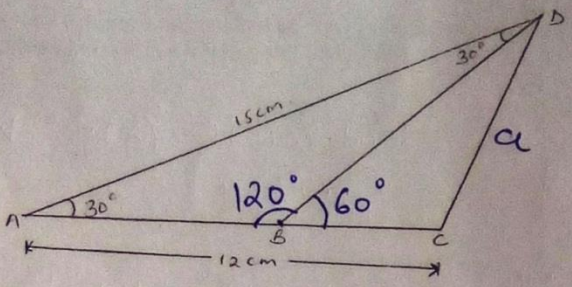
$= \left[\frac{11}{2}t^2 - \frac{2}{3}t^3 - 5t\right]_3^4$ — M1

$= \left(\frac{11}{2} \times 4^2 - \frac{2}{3}(4)^3 - 5(4)\right) - \left(\frac{11}{2}(3)^2 - \frac{2}{3}(3)^3 - 5(3)\right)$ — M1

$= 8\frac{5}{6}$ m. — A1

10

21. In the figure below, AC = 12 cm, AD = 15 cm and B is a point on AC, $\angle BAD = \angle ADB = 30^0$



Calculate to 2 d.p.

a) The length of CD. (3 marks)

$a^2 = 12^2 + 15^2 - 2(12 \times 15)\cos 30$ — M1

$\sqrt{a^2} = \sqrt{57.23085464}$ — M1

$a = 7.57\text{cm}.$ — A1

b) The length of AB. (3 marks)

$\frac{15}{\sin 120} = \frac{AB}{\sin 30^\circ}$ — M1 $AB = 8.66\text{cm}.$ — A1

$AB = \frac{15 \sin 30^\circ}{\sin 120^\circ}$ — M1

c) The area of the triangle BCD. (2 marks)

$\frac{1}{2} \times 3.34 \times 8.66 \sin 60^\circ$ — M1

$= 12.52\text{cm}^2$ — A1

d) The size of $\angle BDC$ (2 marks)

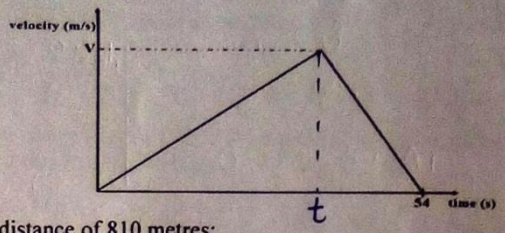
$\frac{3.34}{\sin D} = \frac{7.57}{\sin 60^\circ}$ — M1

$\sin D = 0.382112$

$D = 22.46^\circ$ — A1

10

22. The figure below shows a velocity-time graph of an object which accelerates from rest to a velocity of V ms^{-1} then decelerated to rest in a total time of 54 seconds.



a) If it covered a distance of 810 metres;

i) Find the value of V. (2 marks)

$810 = \frac{1}{2} \times 54 \times V$ — M1

$V = 30\ m/s$ — A1

ii) Calculate its deceleration, given that its initial acceleration was $1\frac{2}{3} ms^{-2}$ (2 marks)

$\frac{30}{t} = \frac{5}{3}$ — M1 $\quad d = \frac{30-0}{54-18}$

$t = 18\ sec$ $\quad d = \frac{5}{6} ms^{-2}$ — A1

b) A bus left town X at 10.45 am and travelled toward town Y at an average speed of 60 km/h. A car left town X at 11.45 am on the same day and travelled along the same road toward Y at an average speed of 100km/h. The distance between town X and town Y is 500km.

i) Determine the time of the day when the car overtook the bus. (3 marks)

Distance covered by bus before car started
$= 60 km/h \times 1h = 60km.$
$RS = 100 km/h - 60 km/h = 40 km/h.$ } M1

Time taken $= \frac{60}{40}$ — M1

$= 1hr\ 30 mins$

Time of the day
$= 11.45 am$
1.30
13.15
$1.15 pm$ or $1315 hr$. — A1

ii) Both vehicles continued towards town Y at their original speeds. Find how long the car had to wait in town Y before the bus arrived. (3 marks)

Time by Bus $= \frac{500}{60} = 8hrs\ 20 min$
Time by Car $= \frac{500}{100} = 5hrs$ } M1

$= 8.20$
5.00 — M1
$3hrs\ 30 Mins$ — A1

10

23. The points A(3, 7), B(5, 5), C(3, 1) and D(1, 5) are vertices of a quadrilateral.

(a) On the grid provided below, plot ABCD on a Cartesian plane (2 marks)

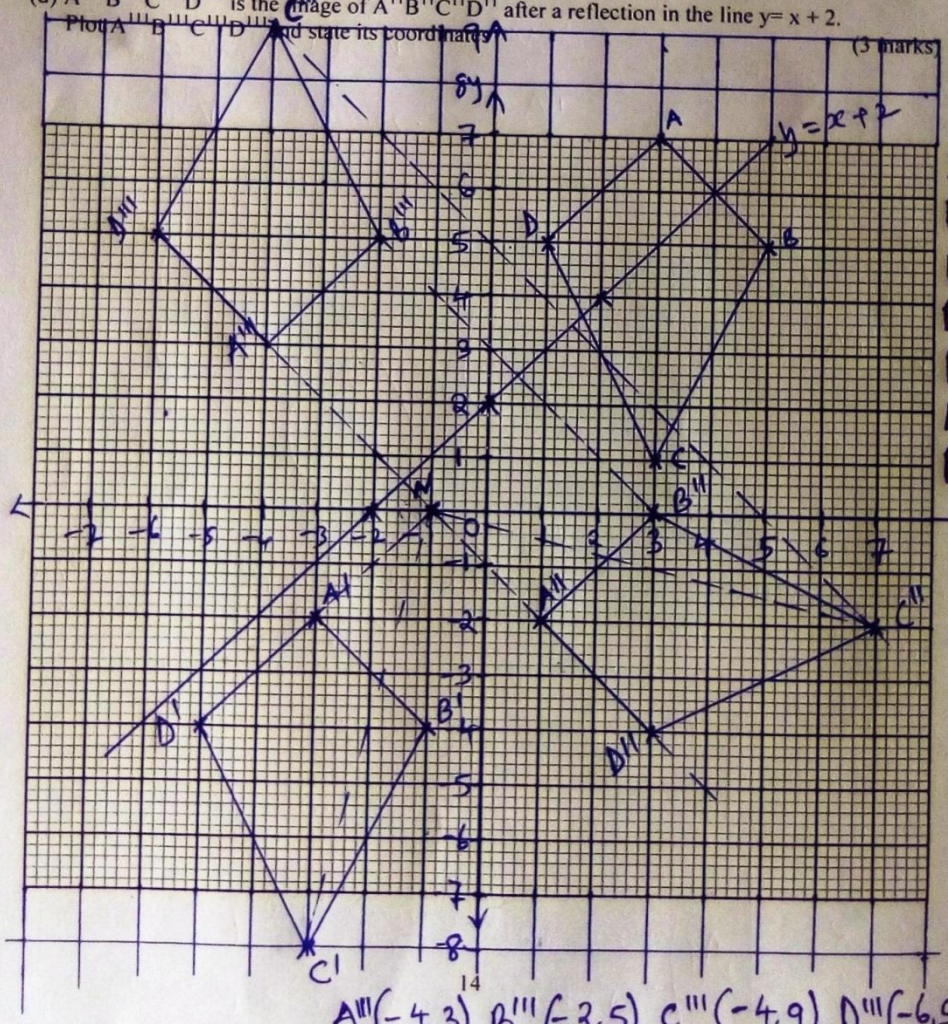
(b) $A^1B^1C^1D^1$ is the image of ABCD under a translational $T = \begin{pmatrix} -6 \\ -9 \end{pmatrix}$. Plot $A^1B^1C^1D^1$ and state its coordinates. (2 marks)

$A^1(-3,-2)$, $B^1(-1,-4)$, $C^1(-3,-8)$, $D^1(-5,-4)$. $-B_1$

(c) Plot $A^{11}B^{11}C^{11}D^{11}$, the image of $A^1B^1C^1D^1$ after a rotation about (-1, 0) through a positive quarter turn. State its coordinates. (3 marks)

$A^{11}(1,-2)$, $B^{11}(3,0)$, $C^{11}(7,-2)$, $D^{11}(3,-4)$. $-B_1$

(d) $A^{111}B^{111}C^{111}D^{111}$ is the image of $A^{11}B^{11}C^{11}D^{11}$ after a reflection in the line $y = x + 2$. Plot $A^{111}B^{111}C^{111}D^{111}$ and state its coordinates. (3 marks)



$A'''(-4,3)$, $B'''(-2,5)$, $C'''(-4,9)$, $D'''(-6,5)$. $-B_1$

S_1 - scale
B_1 - ABCD
B_1 - $A^1B^1C^1D^1$
B_1 - Attempt to rotate
B_1 - $A^{11}B^{11}C^{11}D^{11}$
L_1 - $y = x + 2$
B_1 - $A'''B'''C'''D'''$

10

24. The equation of a curve is; $y = x^2(3-x)$

a) Find;

i) The x - intercept of the curve (2 marks)

$x^2(3-x) = 0$ · $-M_1$ | $3-x=0$

$x^2 = 0$ | $x = 3$.

$x = 0$ | $x = 0$ and 3. A_1

ii) The y -intercept of the curve . (1 mark)

$y = 0^2(3-0)$.

$y = 0$. $- B_1$

b) (i) Determine the stationary points of the curve. (3 marks)

$y = 3x^2 - x^3$.

$\frac{dy}{dx} = 6x - 3x^2 = 0$. $-M_1$ | $2 - x = 0$

$3x(2-x) = 0$ | $x = 2$

$3x = 0$ | $y = 3(2)^2 - (2)^3 = 4$.

$x = 0$ | $(2, 4)$ B_1

$y = 3(0)^2 - (0)^3 = 0$, $(0,0)$ A_1

(ii) For each point in (b)(i) above, determine whether it is a maximum or a minimum. (2 marks)

$(0,0)$

When $x = -1$

$6(-1) - 3(-1)^2 = -9$

When $x = 1$

$6(1) - 3(1)^2 = 3$.

$(0,0)$ is Min. } B_1

$(2,4)$

When $x = 1$

$6(1)^2 - 3(1)^2 = 3$.

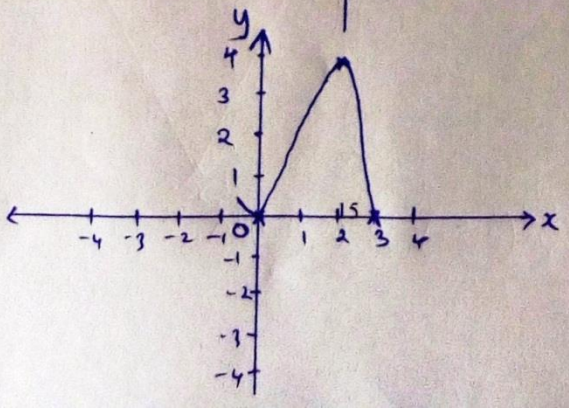
When $x = 3$.

$6(3)^2 - 3(3)^2 = -9$

$(2,4)$ is Max. } B_1.

N/B: B_0 if not determined.

c) Sketch the curve. (2 marks)



B_1 - Plotting the intercepts and stationary points

B_1 - Sketched.

10