Name: ....M/scheme.............. ....Adm No: ....Class:....................

School:...Index No......................................

121/2
MATHEMATICS ALT A
Paper 2

SEPTEMBER 2021
2½ hours

HAWK1 CLUSTER EXAMINATIONS 2021

Kenya Certificate of Secondary Education (KCSE)
MATHEMATICS
PAPER 2
TIME: 2½ HOURS

INSTRUCTIONS TO CANDIDATES

a) Write your Name, School and Index Number in the spaces provided at the top of this page.
b) This paper contains TWO sections: section I and section II
c) Answer all the questions in Section I and strictly any FIVE questions in section II.
d) All answers and working must be written on the question paper in the spaces provided below each question.
e) Show all the steps in your calculations, giving your answers at each stage in the spaces below each question.
f) Marks may be given for correct working even if the answer is wrong.
g) Non-programmable silent electronic calculators and KNEC mathematical tables may be used except where stated otherwise.

FOR EXAMINER'S USE ONLY:

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTAL

SECTION II

17	18	19	20	21	22	23	24	TOTAL

GRAND TOTAL

This paper consists of 15 printed pages. Candidates should check the question paper to ensure that all pages are printed as indicated and no questions are missing

$2.388 + 4 = 2.392$ $\quad 3857 + 11 = 3867$

SECTION 1: (50 MARKS)

Answer ALL the Questions in this section in the spaces provided.

1. Use logarithm tables to evaluate: $\left(\dfrac{24.36 \times 0.847}{361}\right)^{\frac{1}{2}}$ (4 mks)

No.	Std form	Log
24.36	2.436×10^1	1.3867
0.847	8.47×10^{-1}	$\bar{1}.9279 +$
		$\overline{1.3146}$
361	3.61×10^2	$2.5575 -$
		$\bar{2}.7571 \div 2$
0.2392	2.392×10^{-1}	$\bar{1}.3786$

$\frac{\bar{2}}{2} + \frac{0.7571}{2}$

$\bar{1}.3786$

$= 0.2392$

M1 All logs
M1 ± of logs
M1 ÷ 2
A1

2. Given that $x = 8 + \sqrt{2}$ and $y = 2 + \sqrt{2}$ and that $\dfrac{x}{y} = a + b\sqrt{c}$ where a, b and c are integers, find the values of a, b and c. (3 mks)

$\dfrac{8+\sqrt{2}}{2+\sqrt{2}} \cdot \dfrac{(2-\sqrt{2})}{(2-\sqrt{2})}$ ✓M1

$= \dfrac{16 - 8\sqrt{2} + 2\sqrt{2} - 2}{4-2}$

$= \dfrac{14 - 6\sqrt{2}}{2}$

$= 7 - 3\sqrt{2}$ — A1

$\therefore \left.\begin{array}{l} a = 7 \\ b = -3 \\ c = 2 \end{array}\right\}$ B1 (all the values)

3. A rectangle has a length of 35.0 cm and a width of 13 cm. Calculate the percentage error in calculating its area (3 mks)

$\left.\begin{array}{l} \text{max} = 35.05 \times 13.5 = 473.175 \\ \text{min} = 34.95 \times 12.5 = 436.875 \end{array}\right\}$ — M1

Actual $= 35.0 \times 13 = 455$

$A.E = \dfrac{473.175 - 436.875}{2} = 18.15$ — M1

$\%\ \text{error} = \dfrac{18.15}{455} \times 100 = 3.989\%$ — A1.

4. Given that $2\log x^2 + \log\sqrt{x} = k\log x$, find the value of k (3 mks)

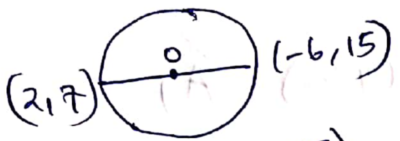
$\log x^4 + \log x^{\frac{1}{2}} = \log x^k$ — M1

$\log(x^4 \cdot x^{\frac{1}{2}}) = \log x^k$ — M1

$x^{4.5} = x^k$

$x = 4.5$ — A1.

5. Write an equation of a circle that has a diameter whose end points are (2, 7) and (-6, 15) in the form $x^2 + y^2 + ax + by + c = 0$ where a, b and c are integers. (4 mks)



$O\left(\frac{2-6}{2}, \frac{7+15}{2}\right)$

$O(-2, 11)$ — B1

Radius $= \begin{pmatrix}-6\\15\end{pmatrix} - \begin{pmatrix}-2\\11\end{pmatrix} = \begin{pmatrix}-4\\4\end{pmatrix}$ ✓M1

$= \sqrt{(-4)^2 + (4)^2} = \sqrt{32}$

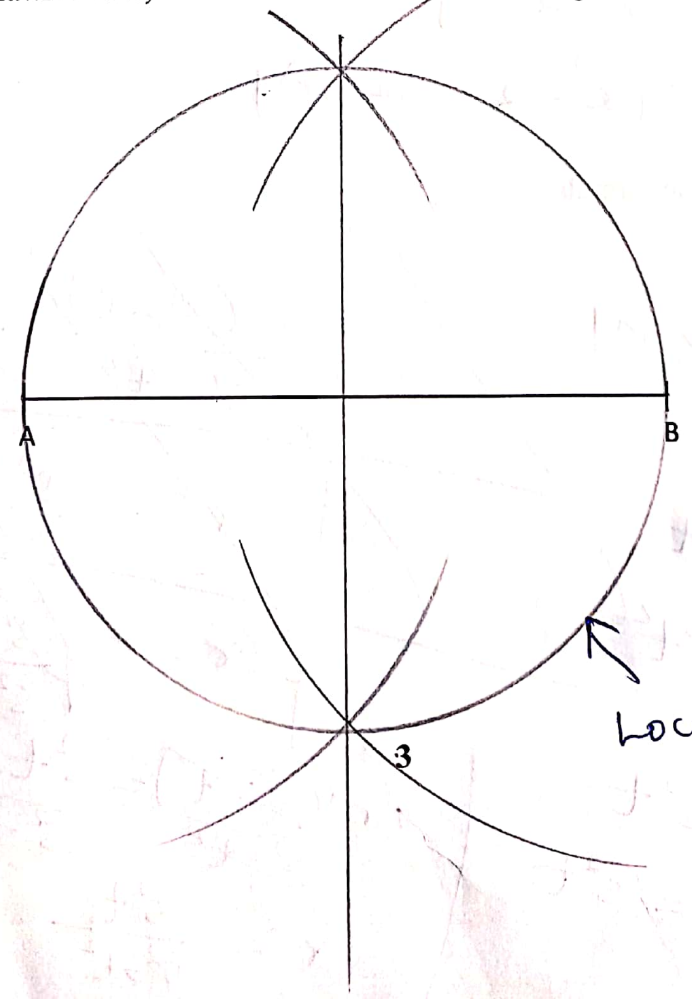
$(x+2)^2 + (y-11)^2 = (\sqrt{32})^2$ — ✓M1

$x^2 + 2x + 2x + 4 + y^2 - 11y - 11y + 121 = 32$

$x^2 + 4x + y^2 - 22y + 93 = 0$

$x^2 + y^2 + 4x - 22y + 93 = 0$ ✓A1

6. On the line AB drawn below, construct locus of P such that angle APB = 90^0. (2 mks)



B1 – Bisecting AB

B1 – Locus P

Locus of P

7. (a) Expand $\left(1-\frac{1}{2}x\right)^5$ up to the term x^3 (1 mk)

$1(1)^5\left(-\frac{1}{2}x\right)^0 + 5(1)^4\left(-\frac{1}{2}x\right)^1 + 10(1)^3\left(-\frac{1}{2}x\right)^2 + 10(1)^2\left(-\frac{1}{2}x\right)^3$

$1 - \frac{5}{2}x + \frac{5}{2}x^2 - \frac{5}{4}x^3$ — B1

(b) Hence use your expansion in (a) above to solve $(0.98)^5$ correct to 4 significant figures (3 mks)

$1-\frac{1}{2}x = 0.98$

$\frac{1}{2}x = 0.02$

$x = 0.04$ ✓B1

$1-\frac{5}{2}(0.04)+\frac{5}{2}(0.04)^2-\frac{5}{4}(0.04)^3$ ✓M1

$= 0.90392$

$= 0.9039$ (4 s.f) ✓A1

8. A quadratic curve passes through pints (-2, 0) and (1, 0). Find the equation of the curve in the form of $y = ax^2 + bx + c$ where a, b and c are constants. (2 mks)

$x = -2$ or $x = 1$

$(x+2) = 0$ $(x-1) = 0$

$y = (x+2)(x-1)$ — M1

$y = x^2 - x + 2x - 2$ — A1

$y = x^2 + x - 2$

9. Make r the subject of the formula (3 mks)

$s = \frac{rt}{\sqrt{r^2 - t}}$

$s^2 = \frac{r^2t^2}{r^2 - t}$ ✓M1

$r^2t^2 = s^2r^2 - s^2t$

$s^2r^2 - r^2t^2 = s^2t$

$\frac{r^2(\cancel{s^2-t^2})}{\cancel{s^2-t^2}} = \frac{s^2t}{s^2-t^2}$ ✓M1

$r^2 = \frac{s^2t}{s^2-t^2}$

$r = \pm\sqrt{\frac{s^2t}{s^2-t^2}}$ ✓A1

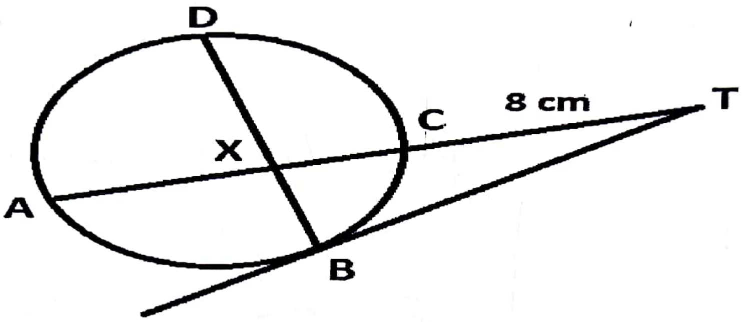
10. Peter bought maize and beans at a cost of sh. 50 and sh. 60 per kg respectively. He then mixed them at a ratio of 2: 3. If he sold the mixture at sh. 70 per kg, find the percentage profit he made. (3 mks)

$$\frac{50(2)+60(3)}{2+3} = 56 \quad \text{— M1}$$

$$70-56 = 14 \quad \text{— M1}$$

$$\frac{14}{56} \times 100 = 25\% \quad \text{— A1}$$

11. In the diagram below, BT is a tangent to the circle at B. AXCT and BXD are straight lines. DB and AC intersect at X. AX = 6 cm, CT = 8 cm, BX = 4.8 cm and XD = 5 cm.



Find the length of BT (3 mks)

$$\frac{x \cdot \cancel{6}}{6} = \frac{4.8 \times 5}{6}$$

$$x = 4\text{ cm} \quad \text{— M1}$$

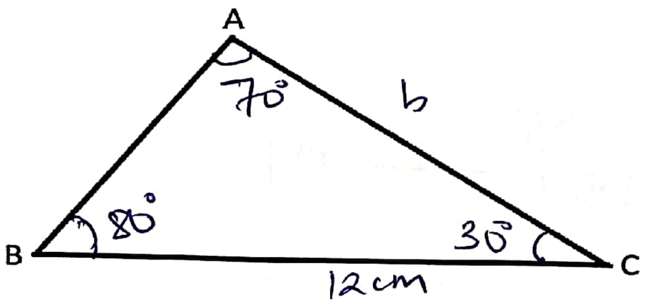
$$AT = 6+4+8 = 18\text{ cm}$$

$$y^2 = 8 \times 18 \quad \checkmark \text{M1}$$

$$y = \sqrt{144}$$

$$y = 12\text{ cm} \quad \checkmark \text{A1}$$

12. In the triangle ABC below BC = 12 cm, angle ABC = 80^0 and angle ACB = 30^0. Calculate correct to 4 significant figures the area of the triangle ABC (3 mks)



$$\frac{b}{\sin 80} = \frac{12}{\sin 70}$$

$$b = \frac{12 \sin 80}{\sin 70} = 12.576 \quad \checkmark \text{M1}$$

$$A = \frac{1}{2} ab \sin C \quad \checkmark \text{M1}$$

$$= \frac{1}{2} \times 12 \times 12.576 \sin 30$$

$$= 37.728$$

$$= 37.73\text{ cm}^2 \ (4\text{ s.f}) \quad \checkmark \text{A1}$$

13. Find the interquartile range of the following data: (3 mks)
2,5,3,2,2,4,7,3,3,6,3,6

2, 2, 2, 3, 3, 3, 3, 4, 5, 6, 6, 7 — B1

$Q_1 = \frac{2+3}{2} = 2.5$
$Q_3 = \frac{5+6}{2} = 5.5$ } — M1

$IQR = 5.5 - 2.5$
$= 3$ ✓ A1

14. A variable P varies directly as T^3 and inversely as the square root of S. When T = 2, S = 9 and P =16. Determine the equation connecting P, T and S hence P when S = 36 and T = 3 (4 mks)

$P \propto \frac{T^3}{\sqrt{S}}$

$P = \frac{KT^3}{\sqrt{S}}$ — M1

$16 = \frac{K(2)^3}{\sqrt{9}}$

$K = \frac{16 \times 3}{8} = 6$

$P = \frac{6T^3}{\sqrt{S}}$ — B1

$P = \frac{6 \times 3^3}{\sqrt{36}}$ — M1

$P = 27$ — A1

15. Amuga bought a plot of land for Ksh. 280,000. After 4 years the value of the plot was Ksh. 495,000. Determine the rate of appreciation per annum correct to one decimal place. (3 mks)

$A = P\left(1+\frac{r}{100}\right)^n$

$\frac{495000}{280,000} = \frac{280,000}{280,000}\left(1+\frac{r}{100}\right)^4$ — M1

$1+\frac{r}{100} = \sqrt[4]{\frac{495000}{280,000}}$

$1+\frac{r}{100} = 1.1531$ — M1

$\frac{r}{100} = 0.1531$

$r = 15.31\%$

$\therefore r = 15.32$ (1d.p) — A1

16. Find the value of x for which the matrix $\begin{pmatrix} 2x & 2 \\ 16 & x \end{pmatrix}$ has no inverse. (3 mks)

$2x^2 - 32 = 0$ —— M1

$\frac{\cancel{2}x^2}{\cancel{2}} = \frac{32}{2}$

$x^2 = 16$ —— M1

$x = \pm 4.$ — A1.

SECTION II (50 MARKS)

(Answer any FIVE questions in this section)

17. The following table shows the rate at which income tax was charged during a certain year.

Monthly taxable income in Kshs.	Tax rate (%)
0 – 9860	10
9861 – 19720	15
19721 – 29580	20
29581 – 39440	25
39441 – 49300	30
49301 – 59160	35
Over 59160	40

A civil servant earns a basic salary of Ksh. 35750 and a monthly allowance of Ksh. 12500. The civil servant is entitled to a personal relief of Ksh. 1062 per month.

Calculate:

a) taxable income (2 mks)

✓m1 ✓A1

= 35750 + 12500 = 48250

b) his net monthly tax (5 mks)

9860 × 0.1 = 986 } – m1
9860 × 0.15 = 1479 }
9860 × 0.2 = 1972 } – m1
9860 × 0.25 = 2465 }
8810 × 0.3 = 2643 – m1
Gross tax = 9545 – m1
Less personal relief 1062 –
Net tax = 8,483 – A1

c) Apart from the salary, the following deductions are also made from his monthly income:

WCPS at 2% of the basic salary
Loan repayment Ksh. 1325
NHIF Ksh. 480

Calculate his net monthly earning (3 mks)

Deductions = $(8483 + \frac{2}{100} \times 35750 + 1325 + 480)$
= 8483 + 715 + 1325 + 480 = 11,003 ✓m1

Net salary = 48250 − 11003 ✓m1
= 37,247 ✓A1

10

18. Two fair dice, one a regular tetrahedron (4 faces) and the other a cube (6 faces) are thrown and the scores are added together.

(a) Complete the table below to show all the possible outcomes (2 mks)

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10

(b) Find the probability that:

i) the sum is 6 (2 mks)

$P(6) = \frac{4}{24}$ ✓M1

$= \frac{1}{6}$ ✓A1

ii) the sum is an odd number (2 mks)

$P(3)$ or $P(5)$ or $P(7)$ or $P(9)$

$\frac{2}{24} + \frac{4}{24} + \frac{4}{24} + \frac{2}{24} = \frac{12}{24}$ ✓M1

$= \frac{1}{2}$ ✓A1

iii) the sum is 6 or 9 (2 mks)

$P(6)$ or $P(9)$

$\frac{4}{24} + \frac{2}{24} = \frac{6}{24}$ ✓M1

$= \frac{1}{4}$ ✓A1

iv) the sum is at least 5 (2 mks)

$P(5)$ or $P(6)$ or $P(7)$ or $P(8)$ or $P(9)$ or $P(10)$

$\frac{4}{24} + \frac{4}{24} + \frac{4}{24} + \frac{3}{24} + \frac{2}{24} + \frac{1}{24} = \frac{18}{24}$ ✓M1

$= \frac{3}{4}$ ✓A1.

10

19. a) A triangle ABC has vertices A (1,4), B (-2,0) and C (4,-2). On the grid provided, draw ABC (1 mk)

b) $A^IB^IC^I$ is the image of ABC after transformation $N = \begin{pmatrix} 3 & 1 \\ 4 & 0 \end{pmatrix}$. Draw $A^IB^IC^I$ on the same grid (2 mks)

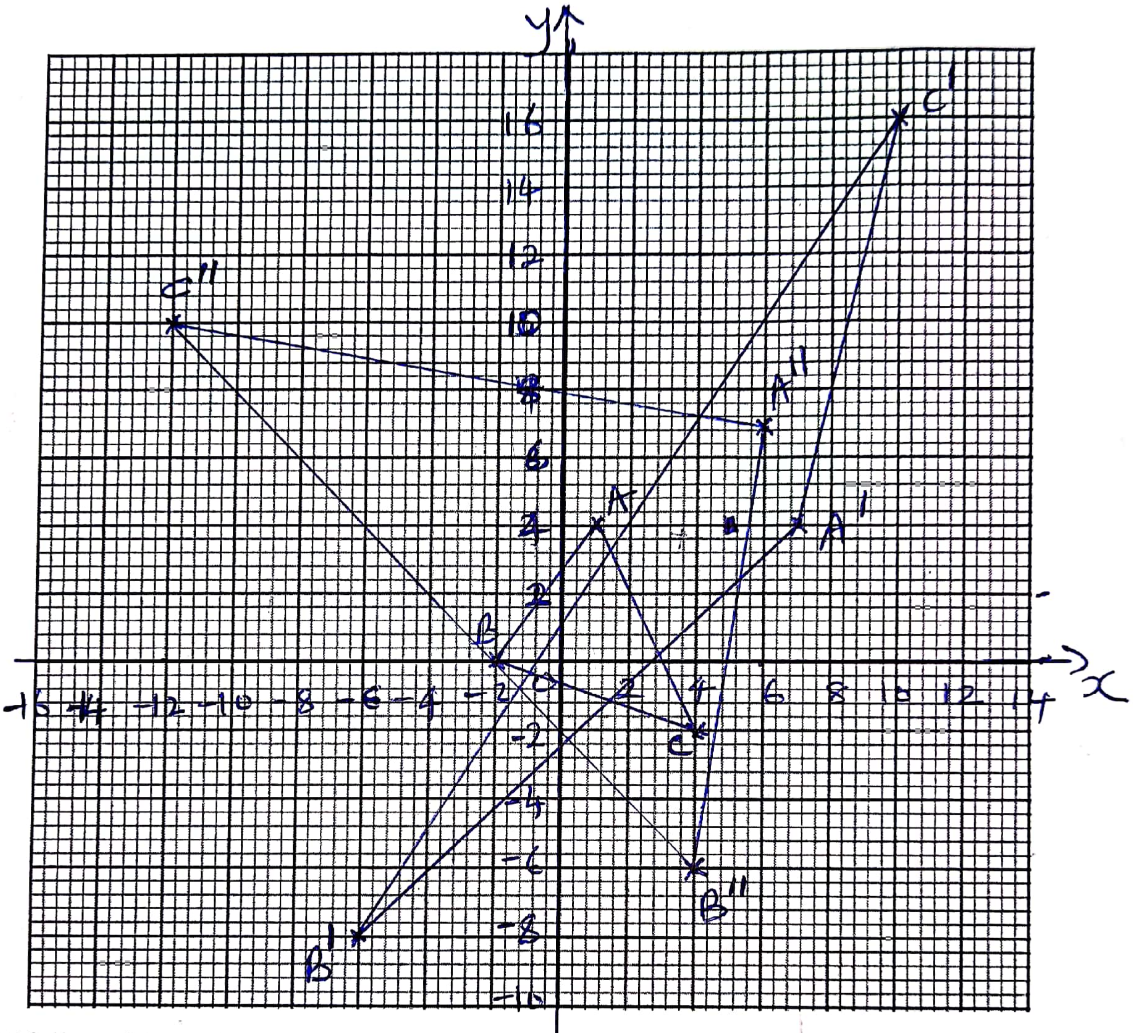
$$\begin{pmatrix} 3 & 1 \\ 4 & 0 \end{pmatrix} \begin{array}{c} A\ B\ C \\ \begin{pmatrix} 1 & -2 & 4 \\ 4 & 0 & -2 \end{pmatrix} \end{array} = \begin{array}{c} A^1\ B^1\ C^1 \\ \begin{pmatrix} 7 & -6 & 10 \\ 4 & -8 & 16 \end{pmatrix} \end{array} - B1$$

c) $A^{II}B^{II}C^{II}$ is the image of $A^IB^IC^I$ after transformation $M = \begin{pmatrix} 2 & -2 \\ 1 & 0 \end{pmatrix}$. Draw $A^{II}B^{II}C^{II}$ and find its coordinates (3 mks)

$$\begin{pmatrix} 2 & -2 \\ 1 & 0 \end{pmatrix} \begin{array}{c} A^1\ B^1\ C^1 \\ \begin{pmatrix} 7 & -6 & 10 \\ 4 & -8 & 16 \end{pmatrix} \end{array} = \begin{array}{c} A^{11}\ B^{11}\ C^{11} \\ \begin{pmatrix} 6 & 4 & -12 \\ 7 & -6 & 10 \end{pmatrix} \end{array} - M1$$

$A^{11}(6,7)$ $B^{11}(4,-6)$ $C^{11}(-12,10)$ - A1

ΔABC - B1
$\Delta A^1B^1C^1$ - B1
$\Delta A^{11}B^{11}C^{11}$ - B1



d) N followed by M is represented by matrix K. Determine K (2 mks)

$K = MN$

$K = \begin{pmatrix} 2 & -2 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 4 & 0 \end{pmatrix} = \begin{pmatrix} -2 & 2 \\ 3 & 1 \end{pmatrix}$ ✓M1 ✓A1

e) If an object of area 4 cm^2 is transformed using matrix K, find the area of the image (2 mks)

$\det = -2 - 6 = -8$ ✓M1

$ASF = \frac{\text{Image area}}{\text{object area}}$

$8 = \frac{x}{4}$

$x = 32\ cm^2$ ✓A1

$\overline{10}$

20. The data below shows the masses in grams of 50 passion fruits

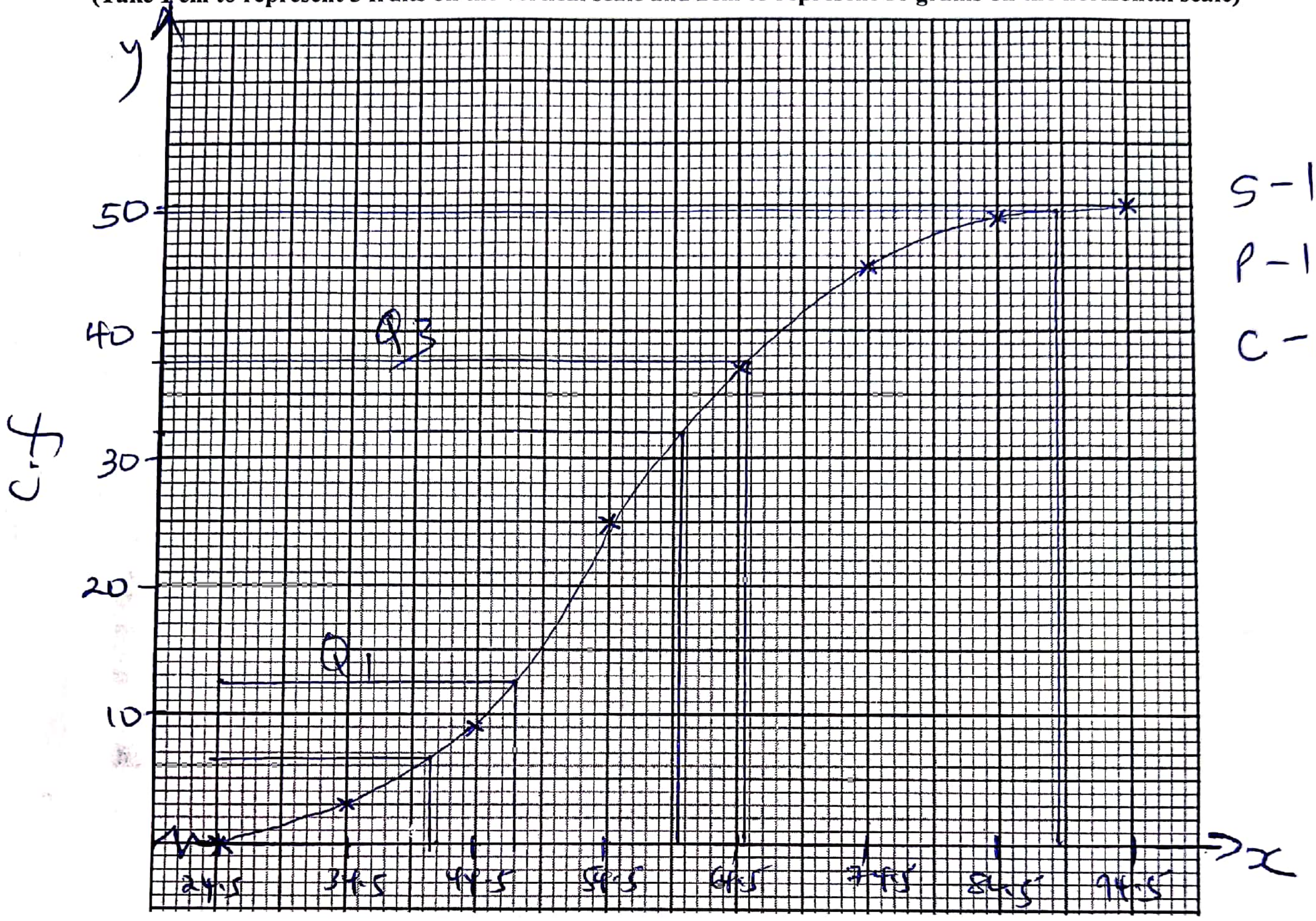
34.5 44.5 54.5 64.5 74.5 84.5 94.5

Mass (g)	25-34	35-44	45-54	55-64	65-74	75-84	85-94
No. of passion fruits	3	6	16	12	8	4	1

C.F 3 9 25 37 45 49 50 B1

a) On the grid provided, draw a cumulative frequency curve for the data (4 mks)

(Take 1 cm to represent 5 fruits on the vertical scale and 2cm to represent 10 grams on the horizontal scale)



S-1

P-1

C-1

b) Use the graph in (a) above to determine:

i) The 64[th] percentile (1 mk)

$\frac{64}{100} \times 50 = 32 \Rightarrow 60 (\pm 1)$ B1

ii) the quartile deviation (3 mks)

$Q_1 = \frac{50}{4} = 12.5 \Rightarrow 47.5$ ✓B1

$Q_3 = \frac{3}{4} \times 50 = 37.5 \Rightarrow 65$ ✓B1

$Q.D = \frac{65 - 47.5}{2}$

$= 8.75$ ✓A1

iii) The percentage of passion fruits whose masses lie in the range 41g to 89g (2 mks)

$41g \Rightarrow 6.5$

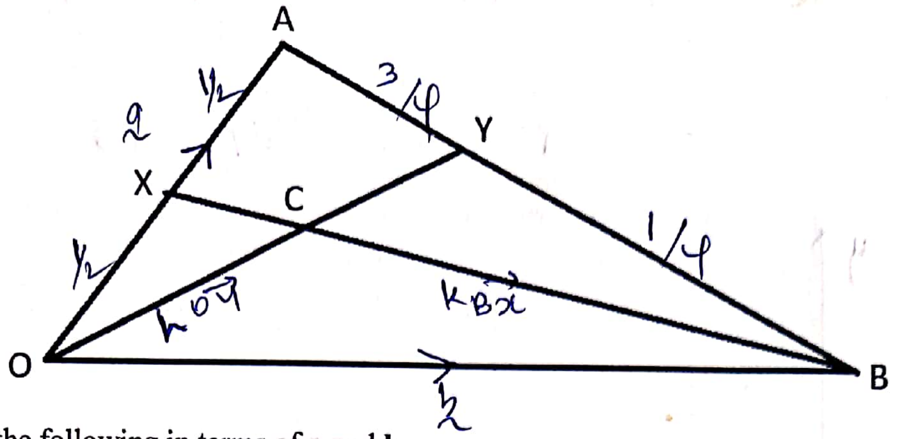
$89g \Rightarrow 49.5$

$49.5 - 6.5 = 43$ — M1

$\frac{43}{50} \times 100 = 86\%$ — A1

10

21. In the figure below, **OA** = **a**, **OB** = **b** and BX meets OY at C. OX: OA = 1: 2 and BY: YA = 1: 3.



a) Express the following in terms of **a** and **b**

i) **BA** (1 mk)

$-\underset{\sim}{b} + \underset{\sim}{a}$ ✓B1

ii) **OY** (2 mks)

$\underset{\sim}{a} + \frac{3}{4}(-\underset{\sim}{a} + \underset{\sim}{b})$ — m1

$\underset{\sim}{a} - \frac{3}{4}\underset{\sim}{a} + \frac{3}{4}\underset{\sim}{b}$

$= \frac{1}{4}\underset{\sim}{a} + \frac{3}{4}\underset{\sim}{b}$ ✓A1

iii) **BX** (1 mk)

$-\underset{\sim}{b} + \frac{1}{2}\underset{\sim}{a}$ ✓B1

b) Given that **OC** = h**OY** and **BC** = k**BX**, determine the value of h and k. (6 mks)

$\vec{OC} = h(\frac{1}{4}\underset{\sim}{a} + \frac{3}{4}\underset{\sim}{b})$

$= \frac{1}{4}h\underset{\sim}{a} + \frac{3}{4}h\underset{\sim}{b}$ —(i) M1

$\vec{OC} = \underset{\sim}{b} + k(-\underset{\sim}{b} + \frac{1}{2}\underset{\sim}{a})$

$= \underset{\sim}{b} - k\underset{\sim}{b} + \frac{1}{2}k\underset{\sim}{a}$

$= (1-k)\underset{\sim}{b} + \frac{1}{2}k\underset{\sim}{a}$ —(ii) m1

$\vec{OC} = \vec{OC}$

$\frac{1}{4}h\underset{\sim}{a} + \frac{3}{4}h\underset{\sim}{b} = (1-k)\underset{\sim}{b} + \frac{1}{2}k\underset{\sim}{a}$

$\frac{1}{4}h = \frac{1}{2}k$

$h = 2k$ ✓B1

$\frac{3}{4}h = 1 - k$ ✓B1

$\frac{3}{4}(2k) = 1 - k$

$\frac{6k}{4} = 1 - k$

$\frac{10k}{10} = \frac{4}{10}$

$k = 2/5$ ✓A1

$h = 2(2/5)$

$h = \frac{4}{5}$ ✓A1

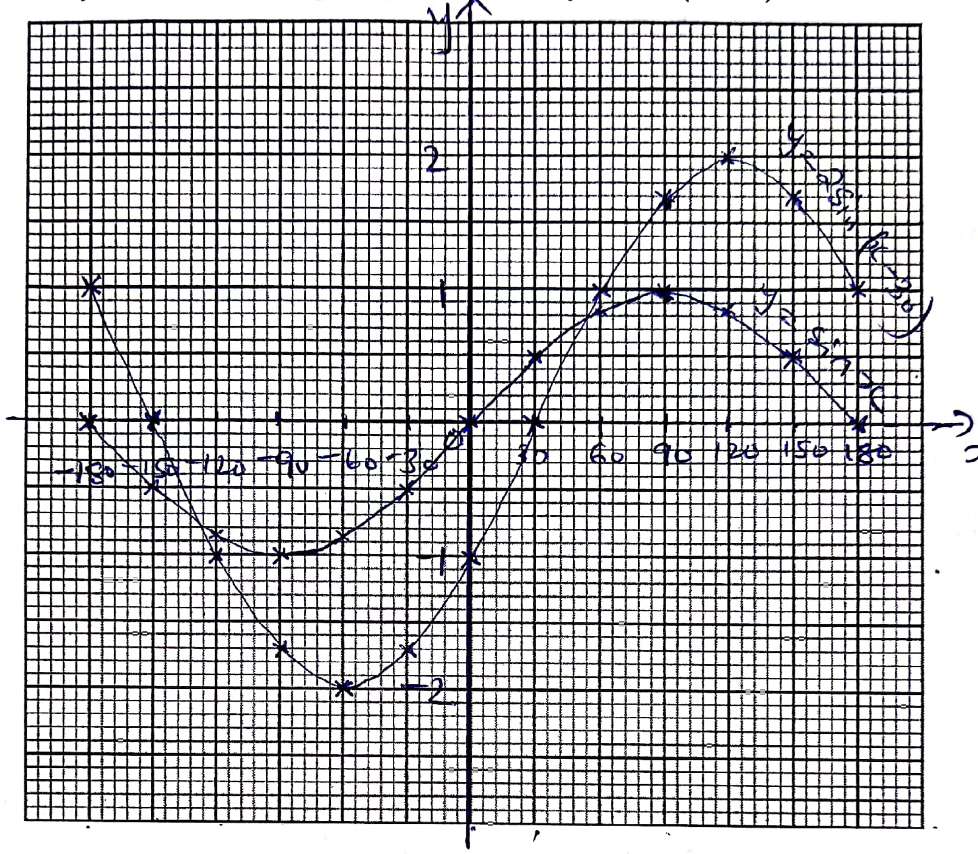
10

22. a) Fill in the table below to 2 decimal places for the graph $y = \text{Sin } x^0$ and $y = 2\text{Sin }(x - 30)^0$ for the range $-180^0 \leq x \leq 180^0$ (2 mks)

x^0	-180^0	-150^0	-120^0	-90^0	-60^0	-30^0	0^0	30^0	60^0	90^0	120^0	150^0	180^0
Sin x^0	0	-0.50	-0.87	-1.00	-0.87	-0.50	0	0.50	0.87	1.00	0.87	0.50	0
2Sin $(x - 30)^0$	1.00	0	-1.00	-1.73	-2.00	-1.73	-1.00	0	1.00	1.73	2.00	1.73	1.00

B2

a) On the grid provided, using a scale of 1 cm represent 30^0 on the x-axis and 1 cm represent 0.5 units on the y-axis, draw the graph of $y = \text{Sin } x^0$ and $y = 2\text{Sin }(x - 30)^0$ on the same axes (5 mks)



S-1
P-1
P-1
C-1
C-1

c) Using your graph,

i) State the amplitude and the period of the graph $y = 2\text{Sin }(x - 30)^0$ (1 mk)

2 units B1

ii) Solve the equation
$\text{Sin } x^0 = 2\text{Sin }(x - 30)^0$ (1 mk)

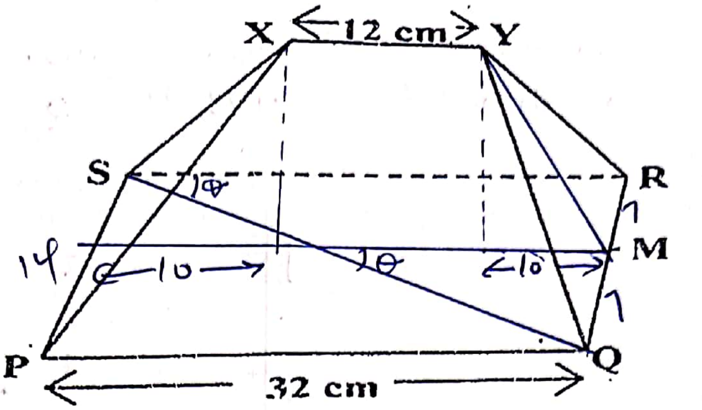
-129° and 54° B1

iii) Describe fully the transformation that will map $y = 2\text{Sin }(x - 30)^0$ on $y = \text{Sin } x^0$ (1 mks)

- A stretch of scale factor $\frac{1}{2}$ parallel to the y-axis followed by a translation $\begin{pmatrix} -30 \\ 0 \end{pmatrix}$. B1.

10

23. The figure below shows a model of a roof with a rectangular base PQRS. PQ =32cm and QR =14cm. The ridge XY = 12 cm and is centrally placed. The faces PSX and QRY are equilateral triangles. M is the midpoint of QR.



Calculate:

(a) i) the length of YM (2mk)

$h = \sqrt{14^2 - 7^2} = \sqrt{196 - 49} = \sqrt{147}$ ✓M1

$= 12.12$ cm ✓A1

ii) the height of Y above the base PQRS (3 mks)

$h = \sqrt{12.12^2 - 10^2}$ ✓M1

$= \sqrt{146.89 - 100}$

$= \sqrt{46.89}$ ✓M1

$= 6.848$ cm ✓A1

(b) The angle between the planes RSXY and PQRS (3 mks)

$\tan\theta = \frac{6.848}{7}$ ✓M1

$\theta = \tan^{-1} 0.9783$ ✓M1

$\theta = 44.37°$ ✓A1

(c) The acute angle between the lines XY and QS (2mks)

$\tan\theta = \frac{14}{32}$ OR $\tan\theta = \frac{7}{16}$ ✓M1

$\theta = \tan^{-1} 0.4375$

$\theta = 23.629°$ ✓A1

10

24. The n^{th} term of a sequence is given by $5n + 1$

(a) i) Write down the first four terms of the sequence (2 mks)

$1^{st} \Rightarrow 5(1)+1 = 6$
$2^{nd} \Rightarrow 5(2)+1 = 11$ } B1
$3^{rd} = 5(3)+1 = 16$
$4^{th} = 5(4)+1 = 21$ } B1

ii) Find the sum of the first forty terms of the sequence (3 mks)

$a = 6 \qquad d = 11 - 6 = 5$

$S_{40} = \frac{40}{2}(2(6) - 39(5))$ ✓ M1 ✓ M1

$= 20 \times 207$

$= 4140$ ✓ A1

(b) The first, fifth and seventh terms of an arithmetic progression (AP) correspond to the first three consecutive terms of a geometric progression (GP). The first term of each progression is 64, the common difference of the AP is **d** and the common ratio of the GP is **r**.

i) Write two equations involving **d** and **r** (2 mks)

$\frac{64}{4} + \frac{4d}{4} = \frac{64r}{4}$

$16 + d = 16r$ ✓ B1

$\frac{64}{2} + \frac{6d}{2} = \frac{64r^2}{2}$

$32 + 3d = 32r^2$ ✓ B1

ii) Find values of **d** and **r** (3 mks)

$d = 16r - 16$

$32r^2 - 3(16r - 16) = 32$ —— M1 (substitution)

$32r^2 - 48r + 48 = 32$

$\frac{32r^2}{16} - \frac{48r}{16} + \frac{16}{16} = \frac{0}{16}$

$2r^2 - 3r + 1 = 0$ —— M1 (forming quadratic equation)

$r = \frac{3 \pm \sqrt{9-8}}{4}$

$r = \frac{3 \pm 1}{4}$

$r = 1$ or $\frac{1}{2}$

when $r = 1$

$d = 16(1) - 16 = 0$

when $r = \frac{1}{2}$

$d = 16(0.5) - 16 = -8$

$\therefore r = 1$ or $\frac{1}{2}$, $d = 0$ or -8 } A1 (all values)