FORM FOUR PAPER II

MARKING SCHEME.

Name.. ADM No ....

School ....Candidate's Signature ....

Date...

121/2
MATHEMATICS
FORM FOUR.
NOVEMBER 2021.
TIME: 2 ½ HOURS

URANGA MATHEMATICS ASSOCIATION - 2021.

Kenya Certificate of Secondary Education (K.C.S.E)

SECOND TERM JOINT EVALUATION.

121/2
Mathematics
Time: 2 ½ Hours

INSTRUCTIONS TO THE CANDIDATES.

- *Write **your name** and **adm number** in the spaces provided above*
- *This paper contains two sections; **Section I** and **Section II**.*
- *Answer all the questions in **section I** and only **five** questions from **Section II***
- *All workings and answers must be written on the question paper in the spaces provided below each question.*
- *Marks may be given for correct working **even if** the answer is wrong.*
- *Non programmable silent electronic calculators and KNEC Mathematical tables may be used **EXCEPT** where stated otherwise.*
- *Show all the steps in your calculations, giving your answers at each stage in the spaces below each question.*
- ***This paper consists of 16 printed pages.***
- ***Candidates should check carefully to ascertain that all the pages are printed as indicated and no questions are missing.***

FOR EXAMINER'S USE ONLY

Section I

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
Marks																	

Section II

Question	17	18	19	20	21	22	23	24	Total
Marks									

GRAND TOTAL

SECTION I (50 MARKS)

Answer All the Questions in this section.

1. A commercial plot is valued at sh. 500,000. The plot depreciates at rate of 10% per six months for a period of 2 years. It then appreciates at a rate of 4% per quarter yearly for three years. Find the value of the plot after 5 years to nearest shillings (4 marks)

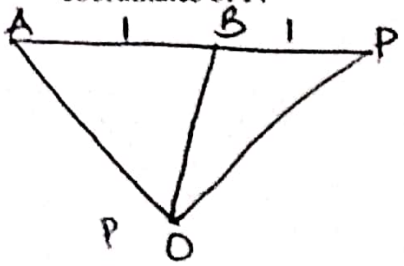
$A = 500{,}000\left(1-\frac{10}{100}\right)^{4}$ — M

$= 328\,050$ — A1

$A = 32805\left(1+\frac{4}{100}\right)^{12}$ — M1

$= 525\,219$ — A1

2. OA = $2i + 3j + 4k$ while OB = $5i + 9j - 2k$. P divides AP externally in the ratio 2:1. Find the coordinates of P. (3 marks)



$\frac{1}{2}\begin{pmatrix}x\\y\\z\end{pmatrix} + \frac{1}{2}\begin{pmatrix}2\\3\\4\end{pmatrix} = \begin{pmatrix}5\\9\\-2\end{pmatrix}$ — M

$\frac{1}{2}x = 4$

$x = 8$

$\frac{1}{2}y = \frac{15}{2}$

$y = 15$

$\frac{1}{2}z = -4$

$z = -8$ — M1

co-ordinates of Point P(8, 15, −8) — A1

3. Rationalize the denominator and simplify (3 marks)

$$\frac{2\sqrt{5}}{\sqrt{5}+2}$$

$\frac{(2\sqrt{5})}{(\sqrt{5}+2)}\frac{(\sqrt{5}-2)}{(\sqrt{5}-2)}$ — M1

$\frac{10-4\sqrt{5}}{5-2}$ — M1

$\frac{10-4\sqrt{5}}{3}$ — A1

4. Make x the subject of the formula. $P = \frac{x^{1/2}y}{x^{1/2} - y}$ (3 marks)

$Px^{1/2} - Py = x^{1/2}y$ — M_1

$Px^{1/2} - x^{1/2}y = Py$

$x^{1/2}(P-y) = Py$ — M_1

$x^{1/2} = \frac{Py}{P-y}$

$x = \left(\frac{Py}{P-y}\right)^2$. — A_1

5. Use logarithms to evaluate correct to 4 s.f (4 marks)

$$\left(\frac{54.5221 - 0.3521}{Tan\,24.8 \times \cos 78}\right)^{1/2}$$

No	Log	
54.17	1.7338	M_1
Tan 24.8	$\bar{1}.6647$ +	M_1
cos 78	$\bar{1}.3179$	
	$\bar{2}.9826$	M_1
	$2.7512 \div 2$	
2.375×10^1	1.3756	

$= 23.75$ — A_1

6. Given that $y = -3 \sin(\frac{2}{5}x + 30)^0$ for $0^0 \leq x \leq 360$. Determine:

a) Amplitude of the curve. (1 mark)

$= 3$ — B_1

b) Phase angle of the curve (1 mark)

$= 30^\circ$ — B_1

b) Period of the curve. (1 mark)

$360 \div 2/5 = 900^\circ$ — B_1

7. Without using logarithm tables solve the equation

$\log(5x - 4) = \log(x + 2) + \frac{1}{3}\log 27.$ (3 marks)

$\log(5x-4) - \log(x+2) = \log 3$ —— M1

$\frac{5x-4}{x+2} = 3$ —— M1

$5x - 4 = 3x + 6$

$x = 5$ —— A1

8. A machine A can do a piece of work in 5 hrs while machine B can do the same amount of work in 8 hours, machine A was set to do the piece of work but after 3 hours. It broke down and machine B did the rest of work. Calculate the time machine B took to do the rest of work. (3marks)

Machine A

Work done in 1hr $= \frac{1}{5}$

In 3 hrs $= \frac{3}{5}$

Remainder $= 2/5$ —— M1

Machine B

Work done in 1hr $= \frac{1}{8}$

In 1hr $= \frac{2}{5}$

$\frac{2}{5} \times 1 \times \frac{8}{1} = \frac{16}{5}$ —— M1

Time = 3hrs 12minutes —— A1

9. A bag contains 5 blue balls and 3 red balls. A ball is picked at random and replaced. A second ball is then picked. Find the probability that

(a) Both ball are red (1 mark)

P(RR)

$\frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$ —— B1

b) The two balls are of different colours (2 marks)

P(BR) + P(RB)

$\left(\frac{5}{8} \times \frac{3}{8}\right) + \left(\frac{3}{8} \times \frac{5}{8}\right)$ —— M1

$\frac{15}{64} + \frac{15}{64} = \frac{30}{64}$

$\frac{15}{32}$ —— A1

10. (a) Find the first 3 terms in ascending powers of x of $(2-x)^5$ (2marks)

$$\times \left.\begin{matrix} 2^5(-x)^0 + 2^4(-x)^1 + 2^3(-x)^2 \\ 1 \quad\quad 5 \quad\quad 10 \end{matrix}\right\} \text{—— M1}$$

$32 - 80x + 80x^2.$ —— A1

(b) Hence find the value of the constant K, for which the coefficient of x in the expansion of $(K+x)(2-x)^5$ is -8 (2marks)

$(k+x)(2-x)^5$

$32k - 80kx + 80kx^2 + 32x - 80x^2 + 80^3$ —— M1

$\therefore \; -80kx + 32x = -8x$

$-80k + 32 = -8$

$-80k = -40$

$k = \frac{1}{2}.$ —— A1

11. The radius of a circle is measured to the nearest meter as 7m. Calculate the percentage error in the circumference. Leave your answer as a mixed number. (Take $\pi = \frac{22}{7}$). (3 marks)

$C = 2\pi r$ $\quad 7 < \begin{matrix} 7.5 \\ 6.5 \end{matrix}$

maximum Circumference $= 2 \times \frac{22}{7} \times 7.5 = 47.14$

Minimum Circumference $= 2 \times \frac{22}{7} \times 6.5 = 40.86$ —— M1

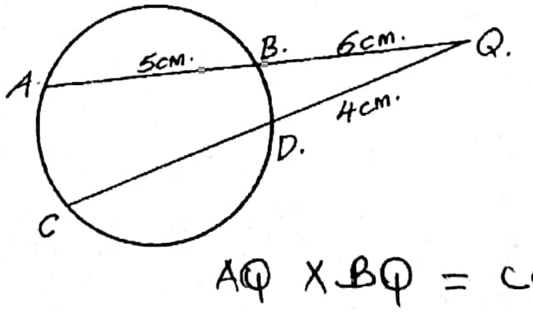
Working Circumference $= 2 \times \frac{22}{7} \times 7 = 44$

Absolute error $= \frac{47.14 - 40.86}{2} = 6.28$ —— M1

% error $= \frac{6.28}{44} \times 100$

$= 14.27\%$ —— A1

12. In the figure below AB and CD are chords of a circle that intersect externally at Q. if AB=5cm, BQ=6cm and DQ=4cm, calculate the length of chord CD (3 marks)



$AQ \times BQ = CQ \times DQ$

$11 \times 6 = (x+4)4$ —— M1

$\frac{66-16}{4} = x$ —— M1

$x = 12.5$ —— A1

7

13. Calculate the variance of the numbers 7, 8, 7, 4, 6, 9, 8 (3marks)

x	f	d = x-7	d²	fd²	fd
4	1	-3	9	9	-3
6	1	-1	1	6	-1
7	2	0	0	0	0
8	2	1	1	2	2
9	1	2	4	4	2
	$\Sigma f = 7$			$\Sigma fd^2 = 21$	$\Sigma fd = 0$

$M_1 - fd^2$

$M_1 - fd$

$\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2 = \text{Variance}$

$\frac{21}{7} - \frac{0}{7} = \text{Variance}$

Variance $= 3$ —— A_1

14. Solve for θ in the equation. $6\cos^2\theta - \sin\theta - 4 = 0$ in the range $0^0 \le \theta \le 180^0$ (3marks)

$\cos^2\theta = 1 - \sin^2\theta$

$6(1-\sin^2\theta) - \sin\theta - 4 = 0$ —— M_1

$6 - 6x^2 - x - 4 - 0$

$6x^2 + x - 2 = 0$

$x = \frac{1}{2}$ or $\frac{-2}{3}$ —— A_1

$\theta = 30°, 150$ —— B_1

15. Solve for x given that the following is a singular matrix $\begin{pmatrix} 1 & 2 \\ x & x-3 \end{pmatrix}$ (2marks)

$1(x-3) - 2x = 0$ —— M_1

$x - 3 - 2x = 0$

$-x = 3$

$x = -3$ —— A_1

16. A circle whose equation is $(x-1)^2 + (y-k)^2 = 10$ passed through point (2,5). Find the coordinates of the two possible centres of the circle. (3marks)

$\begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ k \end{pmatrix} = \begin{pmatrix} 1 \\ 5-k \end{pmatrix}$

$\sqrt{10} = \sqrt{1^2 + (5-k)^2}$ —— M_1

$10 = 1^2 + 25 - 10k + k^2$

$k^2 - 10k + 16 = 0$

$k = 8$ or 2 —— A_1

Centres $(1,8)$ or $(1,2)$ —— B_1

SECTION II (50 MARKS)

Answer ANY FIVE questions in the spaces provided

17. The cost c of producing n items varies directly as n and partly as the inverse of n to produce two items it costs Ksh. 135 and to produce three items it costs Ksh. 140. Calculate.

a) The constant of proportionality and hence write the equation connecting c and n. (5marks)

$C \propto n + \frac{1}{n}$

$C = kn + \frac{T}{n}$ ——— M1

$135 = 2k + \frac{T}{2}$ ——— M1

$140 = 3k + \frac{T}{3}$ ——— M1

$\left.\begin{array}{l} k = 30 \\ T = 150 \end{array}\right\}$ ——— M1

$\therefore\ C = 30n + \frac{150}{n}$ ——— A1

b) The cost of producing 10 items (2marks)

$C = 30(10) + \frac{150}{10}$ ——— M1

$C = 315$ ——— A1

c) The number of items produced at a cost of Ksh. 756. (3marks)

$756 = 30n + \frac{150}{n}$ ——— M1

$30n^2 - 756n + 150 = 0$ ——— M1

$n = \frac{756 \pm 744}{60}$

$n = \frac{1}{5}$ or 25

Number of items $\therefore = 25$ ——— A1

18. The table below shows the height in centimeter of 90 plants in a certain farm.

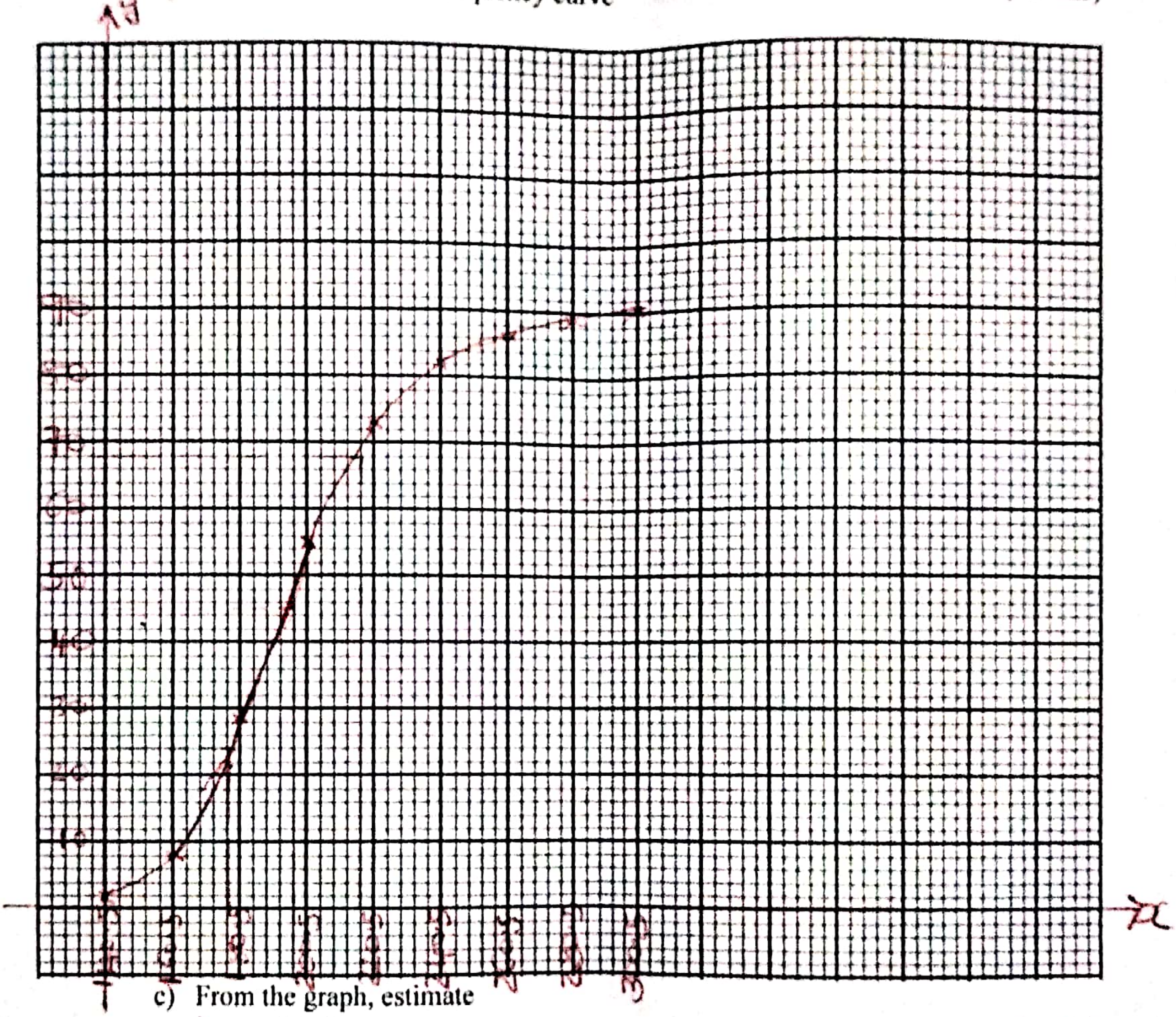
Height	121-140	141-160	161-180	181-200	201-220	221-240	241-260	261-280	281-300
Frequency	2	6	X	26	18	9	4	3	1

a) Find: the value of x (2marks)

$90 - 69$ ——— M1

$= 21$ ——— A1

b) Draw the cumulative frequency curve (4marks)



c) From the graph, estimate

i) Median (1mark)

186.5 cm (± 4) ——— B1

ii) Semi interquartile range (3marks)

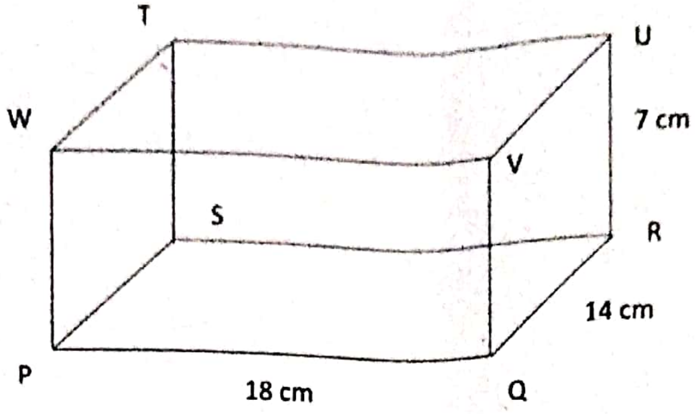
U.Q $= 208.5 \pm 4$

L.Q $= 168.5 \pm 4$ — B1

$\frac{208.5 - 168.5}{2}$ ——— M1

$= 20$ ——— A1

19. The figure below represents a cuboid in which PQ=18cm,QR=14cm and RU=7cm.



a) Name the projection of the line PU on the plane UVWT. (1mark)

WU ——— B_1

b) Calculate correct to 1d.p

i) The size of the angle between PS and QU (2marks)

$\tan\theta = \frac{7}{14}$ ——— M_1

$\theta = \tan^{-1} 7/14$

$\theta = 26.6°$ ——— A_1

ii) The angle between the line QT and the plane PQRS (3marks)

$QS = \sqrt{18^2 + 14^2}$

$= 22.8cm$ ——— B_1

$\theta = \tan^{-1} \frac{7}{22.8}$ ——— M_1

$= 17.1°$ ——— A_1

iii) The angle between planes QWTR and QRUV (2marks)

$\tan\theta = \frac{18}{7}$ ——— M_1

$\theta = \tan^{-1}(18/7)$

$\theta = 68.7°$ ——— A_1

d) point A is the midpoint of TU. Calculate the length QA, correct to 2d.p (2marks)

$QM = \sqrt{14^2 + 9^2}$ ——— M_1

$= 16.64cm$

$QA = \sqrt{16.64^2 + 7^2}$

$QA = 18.1 cm$ ——— A_1

20. A geometric progression (G.P) is such that the product of its first three terms is 8,000.

a) Taking the first term as 'a' and the common ratio as 'r', express 'r' in terms of 'a'. (3marks)

$a, \ ar, \ ar^2$

$a \times ar \times ar^2 = 8000$ —— M1

$a^3 r^3 = 8000$

$r^3 = \frac{8000}{a^3}$ —— M1

$r = \frac{20}{a}$ —— A1

b) The sum of the first three terms in (a) above is 78. Determine the first term and the common ratio of two possible sequences. Hence write the first 6 terms of the two sequences. (5marks)

$a + ar + ar^2 = 78$

$a + a\left(\frac{20}{a}\right) + a\left(\frac{20}{a}\right)^2 = 78$ —— M1

$a^2 - 58a + 400 = 0$

$a = 50$ or 8 —— A1

$r = 2/5$ or $5/2$ —— B1

when $a = 50$, $r = 2/5$

1st 6 terms

$50, 20, 8, 3\frac{1}{5}, 1\frac{7}{25}$ and $\frac{64}{125}$ —— B1

when $a = 8$, $r = \frac{5}{2}$

1st 6 terms

$8, 20, 50, 125, 312\frac{1}{2}$ and $781\frac{1}{4}$ —— B1

c) Find the product of the 8^{th} terms of the two sequences. (2marks)

when $a = 50$ and $r = 2/5$

$8^{th} = 50\left(\frac{2}{5}\right)^7$

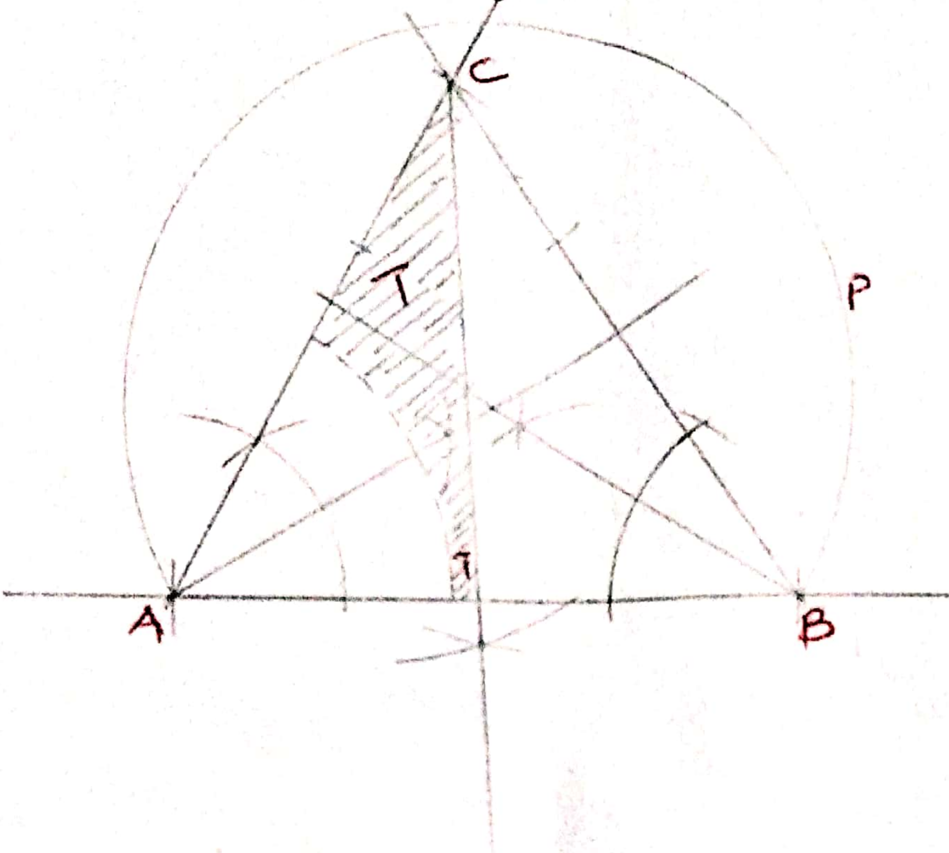
when $a = 8$ and $r = \frac{5}{2}$

8^{th} term $= 8\left(\frac{5}{2}\right)^7$

$50\left(\frac{2}{5}\right)^7 \times 8\left(\frac{5}{2}\right)^7$ —— M1

$= 400$ —— A1

22. (a) Using a ruler and pair of compasses only, construct triangle ABC in which AB = 9cm, AC = 8cm and angle BAC = 60^0. (3marks)



B1 – Line AB drawn
B1 – ∠60° constructed
B1 – Δ ABC completed
B1 – ∠30° constructed
B1 – Centre of the circle
B1 – Locus P drawn
B1 – AT > 4cm shown.
B1 – ∠ACB Bisected
B1 – Region T
B1 – complete dig

(b) On the same side of AB as C, draw the locus of a point such that angle APB = 60^0 (3 marks)

(a) A region T is within the triangle ABC such that AT > 4cm and angle ACT ≥ angle BCT. Show the region T by shading it. (4marks)

23. An air craft leaves town P (30°S, 17°E) and moves directly northwards to Q (60°N, 17° E). It then moved at an average speed of 300 knots for 8 hours westwards to town R.

Determine.

a. The distance PQ in nautical miles (3 marks)

$\theta = 60 + 30 = 90°$ —— B1

$D = 60 \times \theta$

60×90 —— M1

$= 5400nm$ —— A1

b. The position of town R. (3 marks)

$D = 300 \times 8 = 2400nm$ —— B1

$\therefore 2400 = 60 \times \theta \cos 60°$ —— M1

$2400 = 30\theta$

$80° = \theta$

$\theta = x + 17°$

$80 = x + 17$

$x = 63°$

R (60°N 63°W) —— A1

c. The local time at R if local time at Q is 3.12 p.m (2 marks)

$\theta = 80°$

$1° = 4mins$

$80° = ?$

$80 \times 4 = 320 mins$ —— M1

$\Rightarrow$ 5hrs 20mins

+ 3.12
5.20

8.32 pm —— A1

d. The total distance moved from P to R in kilometers (take 1nm = 1.853 km) (2 marks)

$D = 2400nm + 5400nm$ —— M1

$= 7800nm$

7800×1.853

$= 14\,453.4$ KM —— A1.

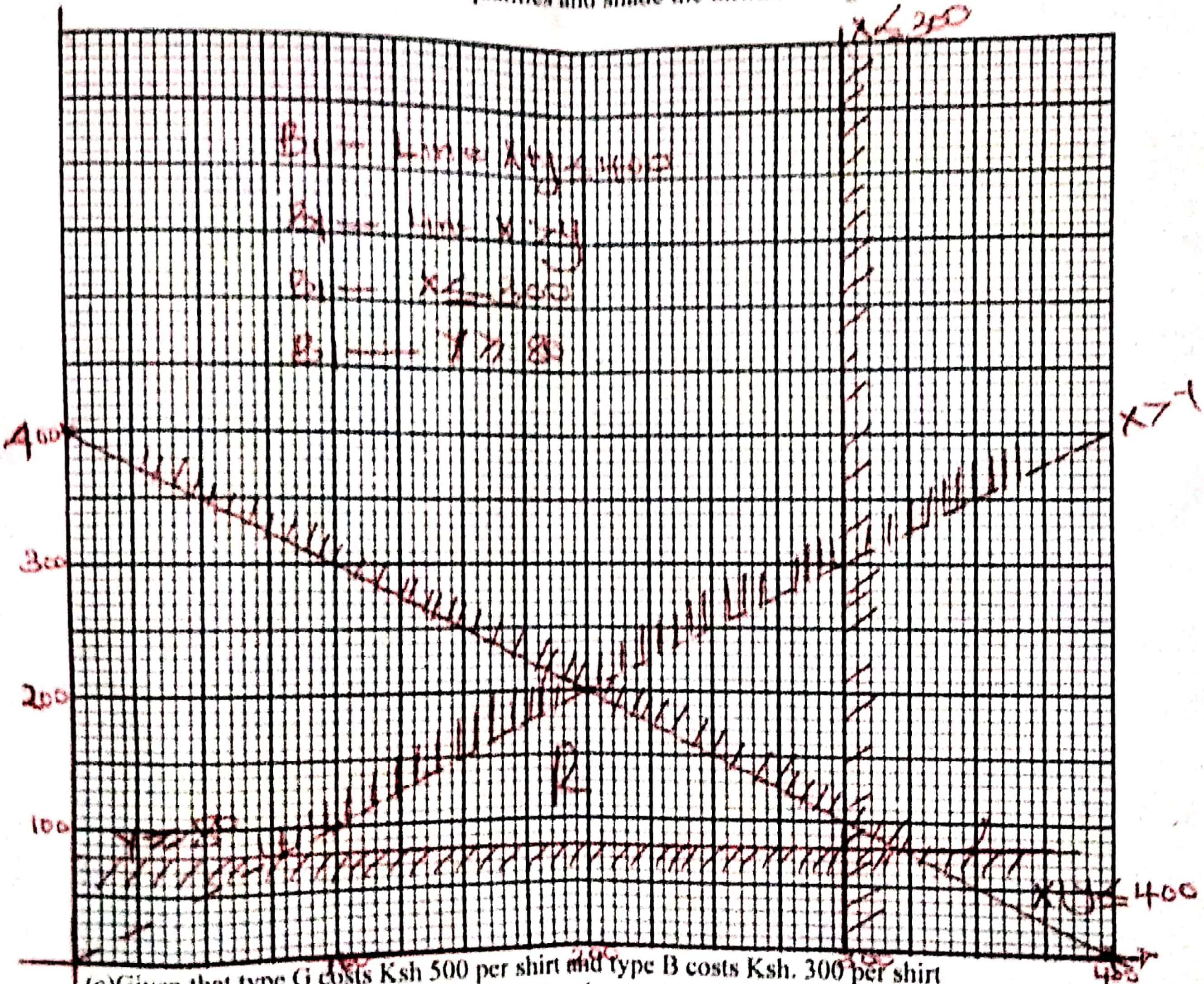
24. A certain uniform supplier is required to supply two types of shirts: one for girls labeled G and the other for boys labeled B. The total number of shirts must not be more than 400. He has to supply more of type G than of type B. However the number of type G shirts must not be more than 300 and the number of type B shirts must not be less than 80. By taking x to be the number of type G shirts and y the number of type B shirts.

(a) Write down in terms of x and y all the inequalities representing the information above. (4 marks)

$x + y \leq 400$ —— B1 $x \leq 300$ —— B1

$x > y$ —— B1 $y \geq 80$ —— B1

(b) On the grid provided draw the inequalities and shade the unwanted regions. (4 marks)



(c) Given that type G costs Ksh 500 per shirt and type B costs Ksh. 300 per shirt

(i) Use the graph in (b) above to determine the number of shirts of each type that should be made to maximize profit. (1 mark)

300 type G and 100 type B —— B1

(ii) Determine the maximum profit (1 mark)

$300 \times 500 + 100 \times 300$ = Ksh 180 000 —— B1