Name.....MARKING SCHEME.....ADM No .....F4 M1.....

School .....Candidate's Signature .....

Date.....

121/1
MATHEMATICS
FORM FOUR.
NOVEMBER 2021.
TIME: 2 ½ HOURS

URANGA MATHEMATICS ASSOCIATION - 2021.

Kenya Certificate of Secondary Education (K.C.S.E)
SECOND TERM JOINT EVALUATION.

121/1
Mathematics
Time: 2 ½ Hours

INSTRUCTIONS TO THE CANDIDATES

- *Write **your name** and **adm number** in the spaces provided above*
- *This paper contains two sections; **Section I** and **Section II**.*
- *Answer all the questions in **section I** and only **five** questions from **Section II***
- *All workings and answers must be written on the question paper in the spaces provided below each question.*
- *Marks may be given for correct working **even if** the answer is wrong.*
- *Non programmable silent electronic calculators and KNEC Mathematical tables may be used **EXCEPT** where stated otherwise.*
- *Show all the steps in your calculations, giving your answers at each stage in the spaces below each question.*
- ***This paper consists of 16 printed pages.***
- ***Candidates should check carefully to ascertain that all the pages are printed as indicated and no questions are missing.***

FOR EXAMINER'S USE ONLY

Section I

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
Marks																	

Section II

TOTAL

GRAND

Question	17	18	19	20	21	22	23	24	**Total**
Marks									

SECTION I (50 marks)

Answer ALL the questions in this section in the spaces provided.

1. Without using a calculator, evaluate; (3 marks)

$\frac{2\frac{1}{5} + \frac{2}{3} \text{ of } 3\frac{3}{4} - 4\frac{1}{6}}{1\frac{1}{4} - 2\frac{2}{3} \div 1\frac{1}{3} + 3\frac{3}{4}}$

Numerator

$\frac{11}{5} + \frac{5}{2} - \frac{25}{6}$

$\frac{47}{10} - \frac{25}{6} = \frac{32}{60}$ M1

$= \frac{8}{15}$

Denominator

$\frac{5}{4} - \frac{8}{3} \times \frac{3}{4} + \frac{15}{4}$

$\frac{5}{4} - 2 + \frac{15}{4} = \frac{20}{4} - 2$

$= 3.$

$\frac{8}{15} \div 3$ M1

$\frac{8}{15} \times \frac{1}{3}$

$= \frac{8}{45}$ A1.

2. The diagonal of a rectangular garden measures $11\frac{1}{4}$ m while its width measures $6\frac{3}{4}$ m. Calculate the perimeter of the garden. (2 marks)

$b = \sqrt{\left(\frac{45}{4}\right)^2 - \left(\frac{27}{4}\right)^2}$

$= \frac{\sqrt{72 \times 18}}{4} = \frac{36}{4} = 9$

$P = 2\left(6\frac{3}{4} + 9\right)$ M1

$= 63/2$

$= 31\frac{1}{2}$ or 31.5 cm. A1.

3. A motorist took 2 hours to travel from one town to another town and 1 hour 40 minutes to travel back. Calculate the percentage change in the speed of the motorist. (3 marks)

Speed to $= \frac{d}{2}$ km/h.

Speed from $= \frac{d}{1\frac{40}{60}}$ km/h. } B1

$= \frac{6}{10}d = \frac{3}{5}d.$

Change in speed $= \frac{3}{5}d - \frac{1}{2}d = \frac{1}{10}d$

% change $= \frac{\frac{1}{10}d}{\frac{1}{2}d} \times 100$ M1 or equivalent

$= \frac{1}{5} \times 100 = 20\%$ increase. A1

4. Given that the ratio x: y = 2:3, find the ratio (5x-2y) : (x+y) (3 marks)

$\frac{x}{y} = \frac{2}{3}.$

$x = \frac{2}{3}y$ M1

$\frac{\left(5\left(\frac{2}{3}y\right) - 2y\right)}{\left(\frac{2}{3}y + y\right)}$ M1

$\frac{\frac{10}{3}y - 2y}{\frac{5}{3}y}$

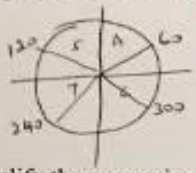
$= \frac{\frac{4}{3}y}{\frac{5}{3}y} = 4:5$ A1.

$2\sin(3x+30) = \sqrt{3}$

5. Give the equation: ~~$\sin(3x+30^0) = \sqrt{3}$~~, for $0^0 \le x \le 90^0$ (3 marks)

$\sin(3x+30) = \sqrt{3}/2$ B1

$3x+30 = 60^0$, B1



$30 \le 3x+30 \le 300$

$3x+30 = 60, 120$ M1

$x = 10, 30$ A1

6. Simplify the expression $\dfrac{a^4 - b^4}{a^3 - ab^2}$ (3marks)

$\dfrac{(a^2+b^2)(a+b)(a-b)}{a(a^2-b^2)}$ B1 B1

$= \dfrac{a^2+b^2}{a}$ B1 accept $a + \dfrac{b^2}{a}$

or $\dfrac{(a^2+b^2)(a+b)(a-b)}{a(a+b)(a-b)}$

$= \dfrac{a^2+b^2}{a}$ or $a + \dfrac{b^2}{a}$.

7. The external length, width and height of an open rectangular container are 41 cm, 21 cm and 15.5cm respectively. The thickness of the material making the container is 5 mm. If the container has 8 litres of water, calculate the internal height above the water level. (4 marks)

$l = 41 - 1.0 = 40$.

$w = 21 - 1.0 = 20$

$h = 15.5 - 0.5 = 15$

$8000\text{cm}^3 = 40 \times 20 \times h$ M1

$h = 10\text{cm}$ A1

height above ...

$= 15 - 10$ M1

$= 5\text{cm}$. A1.

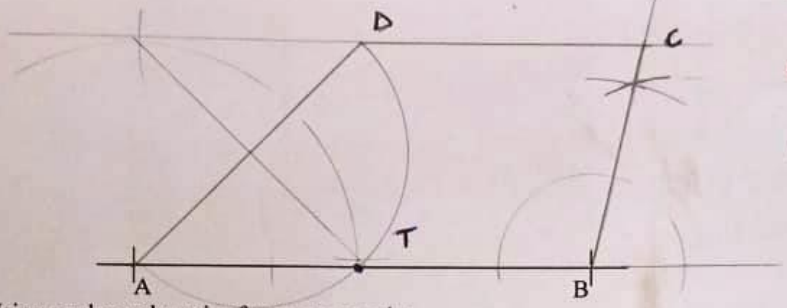
8. Factorise $2x^2y^2 - 5xy - 12$ (2 marks)

$2x^2y^2 - 8xy + 3xy - 12$ M1

$2x(xy-4) + 3(xy-4)$

$= (2x+3)(xy-4)$. A1.

9. Line AB shown below is a side of a trapezium ABCD in which angle ABC= 105^0, BC = 4 cm, CD = 5 cm and CD is parallel to AB.



B_1 ∠ABC ✓

B_1 D located ✓

B_1 ABCD completed ✓

B_1 T located ✓

Using a ruler and a pair of compasses only:

a) Complete the trapezium; (3 marks)

b) Locate point T on line AB such that angle ATD = 90^0 (1 mark)

10. Two matrices A and B are such that $\mathbf{A}=\begin{pmatrix} k & 4 \\ 3 & 2 \end{pmatrix}$ and $\mathbf{B}=\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Given that the determinant of **AB** = 4, find the value of k. (4marks)

$AB = \begin{pmatrix} k & 4 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ M1

$= \begin{pmatrix} 12+k & -2k+16 \\ 9 & 14 \end{pmatrix}$ A_1.

$|AB| \Rightarrow 14(12+k) - 9(2k+16)$ M1

$\Rightarrow 168 + 14k - 18k - 144$

$20 = 4k$

$k = 5$ A_1.

11. A customer paid Ksh 5 880 for a suit after she was allowed a discount of 2% on the selling price. If the discount had not been allowed, the shopkeeper would have made a profit of 20% on the sale of the suit. Calculate the price at which the shopkeeper bought the suit. (3 marks)

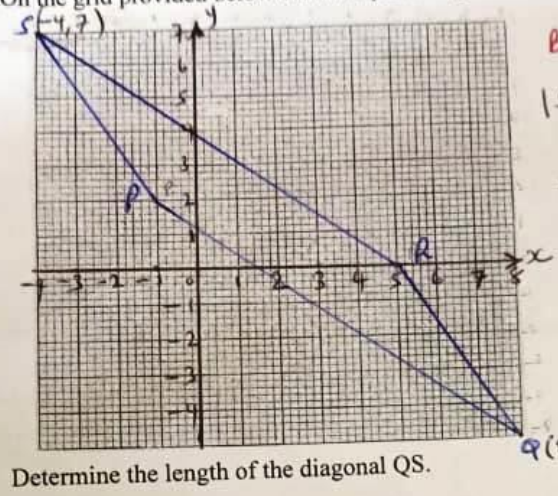
CP = $\frac{100}{98} \times 5880 = 6000$ B_1

$\frac{120}{100} \times 6000$ M1

$= 5000$ A_1.

12. Three vertices of a parallelogram PQRS are P(-1, 2), Q(8, -5) and R (5,0).

(a) On the grid provided below draw the parallelogram PQRS. (1 mark)



B_1 PQRS Drawn ✓

$|SQ| = \left|\begin{pmatrix}+8\\-5\end{pmatrix} - \begin{pmatrix}-4\\7\end{pmatrix}\right| = \left|\begin{pmatrix}12\\-12\end{pmatrix}\right|$

$= \sqrt{12^2 + (-12)^2}$

$= \sqrt{288}$

$= 16.97$ cm. B_1

(b) Determine the length of the diagonal QS. (1 mark)

13. In January, Mambo donated $^1/_6{}^{th}$ of his salary to a children's home while Simba donated $^1/_5{}^{th}$ of his salary to the same children's home. Their total donation for January was Ksh 14 820. In February, Mambo donated $^1/_8{}^{th}$ of his salary to the children's home while Simba donated $^1/_{12}{}^{th}$ of his salary to the children's home. Their total donation for February was Ksh 8 675. Calculate Mambo's monthly salary. (4 marks)

$\frac{1}{6}m + \frac{1}{5}s = 14820$ M_1 expression ✓

$\frac{1}{8}m + \frac{1}{12}s = 8675$ M_1 expression ✓

$m + \frac{6}{5}s = 88920$

$-\quad m + \frac{8}{12}s = 69400$

$\frac{8}{15}s = 19520$ M_1 ✓ Attempt to solve

$s = 36,600$

$m = 88920 - \frac{6}{5}(36,600) = 45,000$ A_1.

14. (a) Express 10500 in terms of its prime factors. (1 mark)

$10500 = 2 \times 2 \times 3 \times 5 \times 5 \times 5 \times 7$. B_1

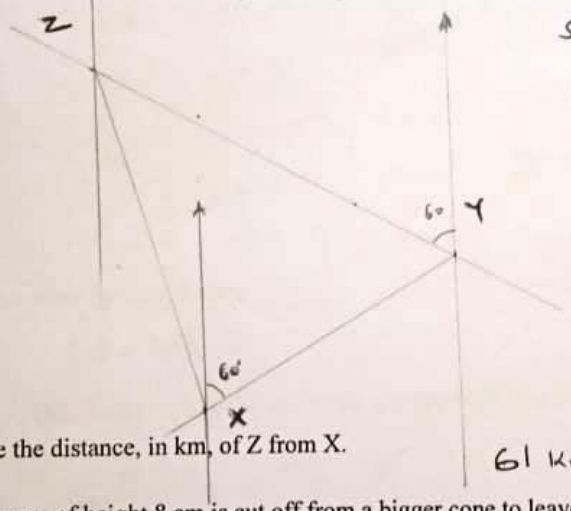
b) Determine the smallest positive number P such that 10500P is a perfect cube. (2 marks)

$P = 2 \times 3 \times 3 \times 7 \times 7$ M_1

$= 882$. A_1.

15. Three police posts X, Y and Z are such that Y is 50 km on a bearing of 060° from X while Z is
70 km from Y and on a bearing of 300° from X.

a) Using a suitable scale, draw a diagram to represent the above situation. (3 marks)



Scale 1cm Rep 10 km S_1

B_1 Y located ✓

B_1 XYZ completed ✓

b) Determine the distance, in km, of Z from X. (1 mark)

61 km (accept 60 - 62 km) B_1

16. A small cone of height 8 cm is cut off from a bigger cone to leave a frustum of height 16 cm. If the volume of the smaller cone is 160 cm^3, find the volume of the frustum. (3 marks)

$\frac{h}{H} = \frac{8}{24}$

lsf $= \frac{1}{3}$.

vsf $= \frac{1}{27}$ B_1

$\frac{160}{V} = \frac{1}{27}$.

$V = 27 \times 160 = 4320$ M_1

$V = 4320 - 160$

$= 4160\ cm^3$. A_1.

SECTION II (50 marks)

Answer any FIVE questions in this section in the spaces provided.

17. The marks scored by a group of pupils in a mathematics test were as recorded in the table below

Marks	Frequency	cf	x	d	fd
			B1	B1	B1
0-9	1	1	4.5	-49.9	-49.9
10-19	2	3	14.5	-39.9	-79.8
20-29	4	7	24.5	-29.9	-119.6
30-39	7	14	34.5	-19.9	-139.3
40-49	10	24	44.5	-9.9	-99.0
50-59	16	40	54.5	0.1	1.6
60-69	20	60	64.5	10.1	202.0
70-79	6	66	74.5	20.1	120.6
80-89	3	69	84.5	30.1	90.3
90-99	1	70	94.5	40.1	40.1
					-33.0

M1, A1.

a) i) State the modal class (1 mark)

60-69 B1

ii) Determine the class in which the median mark lies. (2 marks)

50-59. B1

b) Using an assumed mean of 54.4, calculate the mean mark (7 marks)

$$\bar{x} = 54.4 + \frac{-33}{70} \quad M1$$

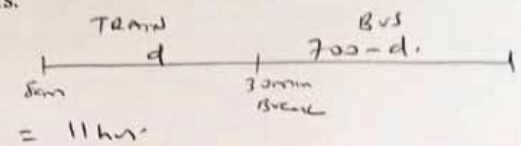
$$= 53.92857$$

$$\simeq 53.93. \quad A1.$$

18. Makau made a journey of 700 km partly by train and partly by bus. He started his journey at 8.00 a.m. by train which travelled at 50 km/h. After alighting from the train, he took a lunch break of 30 minutes. He then continued his journey by bus which travelled at 75 km/h. The whole journey took 11 ½ hours.

a). Determine:

i). the distance travelled by bus; (4 marks)



Time = $11\frac{1}{2}$ hr − 30min = 11 hrs.

$$\frac{d}{50} + \frac{700-d}{75} = 11 \quad M_1$$

$$3d + 1400 - 2d = 150 \times 11 \quad M_1$$

$$d = 250\text{km}.$$

Bus travelled $(700 - 250) = 450\text{km}$ M_1 A_1.

ii). the time Makau started travelling by bus. (3 marks)

Time taken by Bus. $= \frac{250}{50} = 5\text{hrs}.$ B_1

8.00 am + 5hrs + 30min. M_1

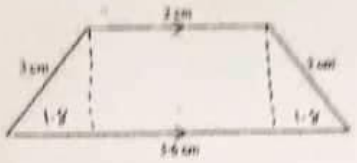
= 1.30 pm. A_1.

b).The bus developed a puncture after travelling 187 ½ km. It took 15 minutes to replace the wheel. Find the time taken to complete the remaining part of the journey. (3 marks)

Remaining distance = $450 - 187.5 = 262.5\text{km}$ M_1.

Time taken = $\frac{262.5}{75} = 3\frac{1}{2}\text{ hrs}.$ M_1 A_1.

19. The diagram below (not drawn to scale) represents the cross- section of a solid prism of height 8.0 cm



$h = \sqrt{3^2 - 1.8^2}$

$= 2.4$ B1

Cross Section Area:

$= \frac{1}{2} \times 2.4 \times 7.6 = 9.12$

(a) Calculate the volume of the prism (3 marks)

Vol = 9.12×8 M1

$= 72.96\ cm^3$. A1.

(b) Given that the density of the prism is $5.75 g/cm^3$, calculate its mass in grams (2 marks)

Mass = 5.75×72.96 M1

$= 419.52\ g$. A1.

(c) A second prism is similar to first one but is made of a different material. The volume of the second is $246.24 cm^3$

(i) Calculate the area of the cross section of the second prism (3 marks)

VSf = $\frac{246.24}{72.96} = \frac{27}{8}$

LSf = $\frac{3}{2}$.

ASf = $\frac{9}{4}$ B1

$A = \frac{9}{4} \times 9.12$ M1

$= 20.52\ cm^2$ A1.

(ii) Given that the ratio of the mass of the first to that of the second is 2:5 and find the density of the second prism (2 marks)

$\frac{2}{5} = \frac{419.52}{M}$

$M = \frac{5}{2} \times 419.52$

$= 1,048.8\ g$ M1

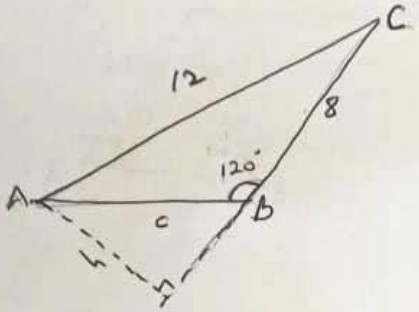
$D = \frac{1048.8}{246.24}$

$= 4.2593$

$\approx 4.259\ g/cm^3$. A1.

20. In a triangle ABC, BC = 8 cm, AC = 12 cm and angle ABC = 120°.

a) Calculate the length of AB, correct to one decimal place. (4 marks)



$c^2 = 12^2 = 8^2 + c^2 - 2(8)(c)\cos 120$ M1

$144 - 64 + 16c\cos 120 = c^2$

$c^2 + 8c - 80 = 0.$

$c = \frac{-8 \pm \sqrt{64+320}}{2}$ M1

$c_1 = 5.798$ A1.

$c_2 = -13.798$ (ignore)

$AB = 5.8$ cm. B1 CAO.

b). If BC is the base of the triangle, calculate, correct to one decimal place:

i). the perpendicular height of the triangle; (2 marks)

$\frac{1}{2} \times 8 \times h = \frac{1}{2} \times 5.798 \times 8 \sin 120.$ M1

$h = 5.798 \sin 120.$

$= 5.0212$

$\doteq 5.0.$ A1 (CAO)

ii). the area of the triangle; (2 marks)

$A = \frac{1}{2} \times 5.798 \times 8 \sin 120.$ M1

$= 20.08$

$= 20.1$ A1 (CAO)

iii). the size of angle ACB. (2 marks)

$\frac{12}{\sin 120} = \frac{5.798}{\sin C}.$ M1

$\sin C = \frac{5.798 \sin 120}{12.}$

$= 0.4184$

$C = 24.736°$

$C = 24.7°$ A1.

21.a) Express $\frac{1}{x-2} - \frac{2}{x+5} = \frac{3}{x+1}$ in the form $ax^2 + bx + c = 0$,

Where a, b and c are constants hence solve for x. (4 marks)

$1(x+5)(x+1) - (2x-4)(x+1) = (3x-6)(x+5)$ M1

$-4x^2 - 5x + 31 = 0$. A1. accept equivalents.

$x = \frac{5 \pm \sqrt{25 + 496}}{-8}$ M1

$x_1 = \frac{5 + 22.825}{-8} = -3.478$

$x_2 = \frac{5 - 22.825}{-8} = 2.228$ } A1.

b). Neema did y test and scored a total of 120 marks. She did two more tests which she scored 14 and 13 marks. The mean score of the first tests was 3 marks more than the mean score for all the tests she did. Find the total number of tests that she did. (6 marks)

$\frac{120}{y} - \frac{147}{y+2} = 3$. M1

$120(y+2) - 147y = 3y(y+2)$ M1

$-3y^2 - 33y + 240 = 0$. M1

$y^2 - 11y - 80 = 0$

$y^2 - 5y + 16y - 80 = 0$

$y(y-5) + 16(y-5) = 0$ M1

$(y-5)(y+16) = 0$

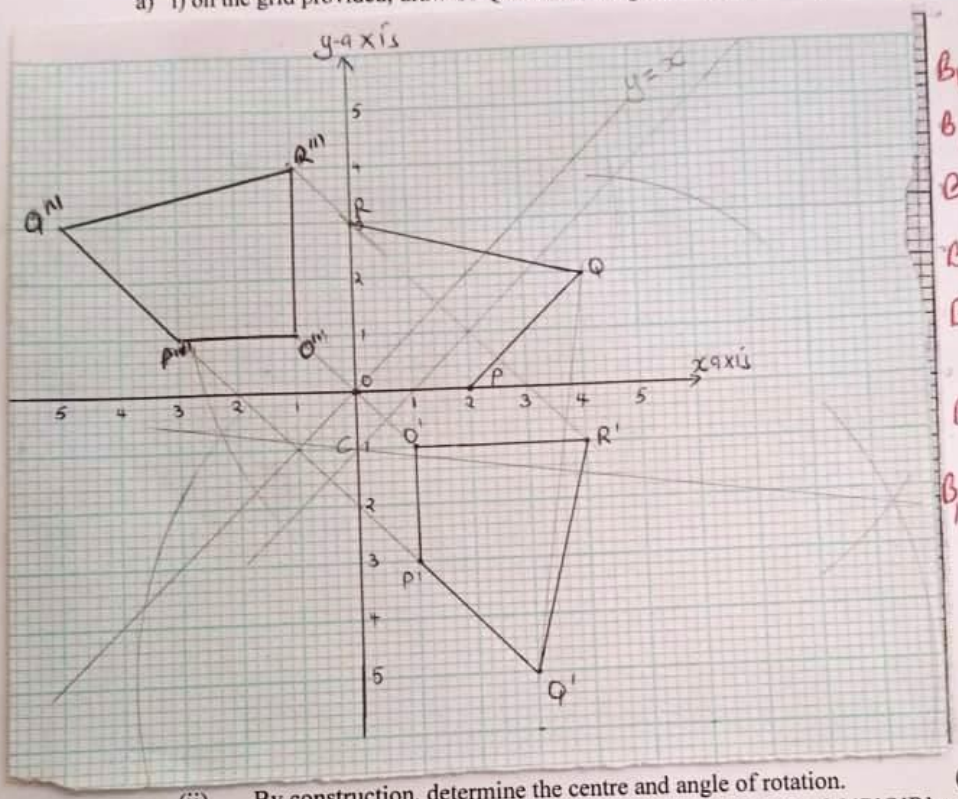
$y = 5$
$y = -16$ } A1.

$y + 2 = 5 + 2 = 7$ tests. B1.

Q (4, 2)

22. The vertices of quadrilateral OPQR are O (0, 0), P(2, 0) and R (0,3). The vertices of its image under a rotation are O' (1, -1), P' (1,-3), Q' (3, - 5) and R' (4, - 1)

a) i) on the grid provided, draw OPQR and its image O'P'Q'R'. (2 marks)



B1 OPQR ✓
B1 O'P'Q'R' ✓
B1 any 2 Bisectors ✓
B1 C located & Stated as (0,-1)
B1 -ve 90° B1 or 270°
B1 Mirror line y=x drawn ✓
B1 O"P"Q"R" ✓

(ii) By construction, determine the centre and angle of rotation. (3 marks)

(b) On the same grid as (a) (i) above, draw O" P"Q"R", the image of Q'P'Q'R' under a reflection in the line y = x. (2 marks)

(c) From the quadrilaterals drawn, state the pairs that are:

(i) Directly congruent; (1 mark)

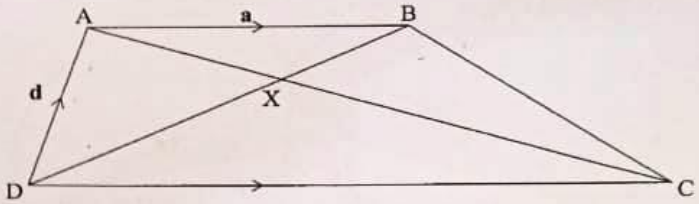
OPQR & O'P'Q'R' B1

(ii) Oppositely congruent. (2 marks)

OPQR & O"P"Q"R" B1

O'P'Q'R' & O"P"Q"R" B1

23. In the figure below, ABCD is a trapezium. AB is parallel to DC, diagonals AC and DB Intersect at X and **DC** = 2 **AB**. **AB** = **a**, **DA** = **d**, **AX** = k **AC** and **DX** = h **DB**, where h and k are constants.



(a) Find in terms of **a** and **d**

(i) **BC;** (2 marks)

$-\underset{\sim}{a} - \underset{\sim}{d} + 2\underset{\sim}{a}$ M1

$= \underset{\sim}{a} - \underset{\sim}{d}$. A1.

(ii) **AX** (2 mark)

$k(AC) = k(2\underset{\sim}{a} - \underset{\sim}{d}) = 2k\underset{\sim}{a} - k\underset{\sim}{d}$. M1 A1

(iii) **DX.** $= h(DB)$ (1 mark)

$= h(\underset{\sim}{d} + \underset{\sim}{a}) = h\underset{\sim}{d} + h\underset{\sim}{a}$. B1

(b) Determine the values of h and k (5 marks)

$DX = \underset{\sim}{d} + 2k\underset{\sim}{a} - k\underset{\sim}{d}$. * $DX = h\underset{\sim}{d} + h\underset{\sim}{a}$

$= \underset{\sim}{d} - k\underset{\sim}{d} + 2k\underset{\sim}{a}$. B1 ✓ expression.

$= (1-k)\underset{\sim}{d} + 2k\underset{\sim}{a}$ M1 factorization ✓

$1 - k = h$. } M1 comparison ✓

$h = 2k$ }

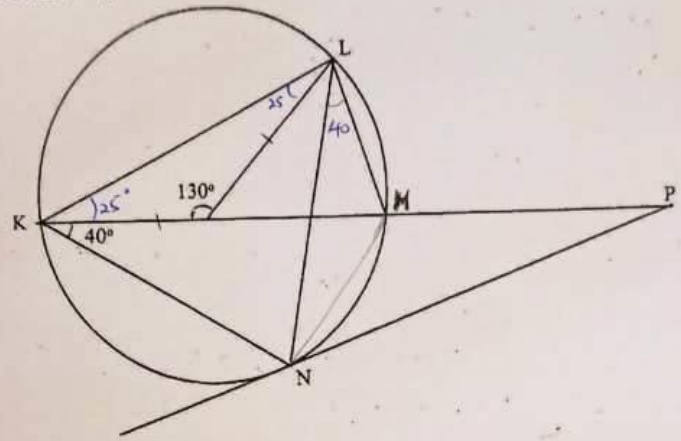
$2k = 1 - k$. M1 attempt ✓

$3k = 1$

$k = 1/3$. } A1. k & h ✓.

$h = 2k = 2/3$ }

24. In the figure below, K M and N are points on the circumference of a circle center O. The points K, O, M and p are on a straight line.PN is a tangent to the circle at N. Angle KOL = 130^0
and angle MKN = 40^0



Find the values of the following angles, stating the reasons in each case:

a) <MLN (2 marks)

40° : Angles subtended by a common chord to the arc in the same segment are equal. B1 B1.

b) <OLN (2 marks)

25° : Diameter subtends right angles at the circumference B1 or Base Angles of Isosceles triangles are equal. B1

c) < LNP (2 marks)

65° : Angles in the alternate segments are equal B1 B1

d) <MPN (2 marks)

40° Angles in the alternate segment are equal B1 B1

e) <LNK (2 marks)

65° : Angles subtended at the circumference are Half the angles subtended by the common chord at the centre, in the same segment. B1 B1.