Name..... M/SCHEME ..... ADM. Number: .....

School: ..... Candidate's Signature .....

121/2
Mathematics Alt. A
FORM THREE.
OCTOBER 2022.
2 ½ Hours.

URANGA MATHEMATICS ASSOCIATION-2022.

Kenya Certificate of Secondary Education
MATHEMATICS
121/2
FORM THREE
TIME: 2 ½ HOURS

INSTRUCTIONS TO CANDIDATES:

- Write your name, school, admission number and sign in the spaces provided above.
- This paper contains **TWO** sections: Section **I** and Section **II**.
- Answer **ALL** the questions in Section **I** and **FIVE** questions from section **II**.
- All answers and working **MUST** be written on the question paper in the spaces provided below each question.
- Marks may be given for correct working even if the answer is wrong.
- Non-programmable silent electronic calculators and KNEC Mathematical tables may be used, except where stated otherwise.

FOR EXAMINERS USE ONLY

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

SECTION II

17	18	19	20	21	22	23	24	Total

Grand Total

This paper consists of 15 printed pages. Candidates should check to ascertain that all pages are printed as indicated and that no questions are missing.

SECTION I (50MARKS)

Answer all the questions in this section in the spaces provided.

1. Calculate the percentage error in the volume of a cone whose radius is 9.0 and height 12.0cm. (3 marks)

Max. vol. $= \frac{1}{3} \times \frac{22}{7} \times 9.05^2 \times 12.05$

$= 1033.92$

Min. vol $= \frac{1}{3} \times \frac{22}{7} \times 8.95^2 \times 11.95$

$= 1002.81$

Actual vol $= \frac{1}{3} \times \frac{22}{7} \times 9^2 \times 12$

$= 1018.29$

A.E $= \frac{1}{2} \times 31.11$ ✓

$= 15.555$

% error $= \frac{15.555}{1018.29} \times 100$ ✓

$= 1.5276\ \%$ ✓

M1 M1 A1 — 03

2. Use logarithms to evaluate

$$\frac{\sqrt{58.32 \times 0.9823}}{693.5}$$

(3 marks)

No.	log
58.32	1.7659 +
0.9823	$\bar{1}.9922$ +
	$1.7581 \times \frac{1}{2}$
	0.8791 —
693.5	2.8410
1.091×10^{-2}	$\bar{2}.0381$
$\Rightarrow 0.01091$	

M1 – All logs ✓

M1 – ✓ + Gud –

A1 – Antilog

03

3. **OA**=3**i** + 4**j** - 6**k** and **OB** = 2**i** + 3**j** + **k**. P divides a line AB in the ratio 3:-2. Write down the co-ordinates of P. (3 marks)

$$\underset{\sim}{OP} = \frac{-2}{1}\begin{pmatrix} 3 \\ 4 \\ -6 \end{pmatrix} + \frac{3}{1}\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 1 \\ 15 \end{pmatrix}$$

$$= (0, 1, 15)$$

M1 M1 A1 — 03

4. Quantity **y** varies partly as the square of **x** and partly as **x** .If **y** = 26 when **x** = 3 and **y** = 14 when **x** = -2, find the equation connecting **y** and **x** (3 marks)

$y = mx^2 + nx$

$9m + 3n = 26$

$4m - 2n = 14$

$m = 47/15$

$n = -11/15$

Equation connecting y and x

$y = 47/15\,x^2 - 11/15\,x$

M1 M1 A1 — 03

5. Write the expression of $\left(2-\frac{1}{5}x\right)^6$ up to the term in x^4. Hence use the expansion to find the value of $(1.96)^6$ correct to 3 decimal places. (4 marks)

$2^6(-1/5x)^0_1 \quad 2^5(-1/5x)^1_6 \quad 2^4(-1/5x)^2_{15} \quad 2^3(-1/5x)^3_{20} \quad 2^2(-1/5x)^4_{15}$ M1

$64 - \frac{192}{5}x + \frac{48}{5}x^2 - \frac{32}{25}x^3 + \frac{12}{125}x^4$ A1

$2 - 1/5\,x = 1.96$

$x = 0.2$

$64 - \frac{192}{5}(0.2) + 48/5\,(0.2)^2 - \frac{32}{25}(0.2)^3 + 12/125\,(0.2)^4$ M1

$= 56.694$ (3 d.p) A1 — 04

6. Solve the equation $3\log_3 X + 4 = \log_3 24$ (3 marks)

$\log_3 x^3 + \log_3 81 = \log_3 24$ M1 - $\log_3 81$

$81x^3 = 24$ M1 - dropping logs

$x^3 = \frac{24}{81} = \frac{8}{27}$

$x = 2/3$ A1

7. Solve the simultaneous equations: (4 marks)

$2x - y = 3$
$x^2 - xy = -4$

$2x - y = 3$
$y = 2x - 3$
$x^2 - x(2x-3) = -4$ ✓
$x^2 - 2x^2 + 3x = -4$
$-x^2 + 3x + 4 = 0$
$x^2 - 4x + x - 4 = 0$ ✓
$x(x-4) + 1(x-4) = 0$

$x = -1$ or $x = 4$
when $x = -1$, $y = -5$
when $x = 4$, $y = 5$

M1 (subst)
M1 (quadratic eqn.)
M1 (values of x)
B1 (pairing values of x and y)
04

8. Find the compound interest on sh. 25,500 invested for 3 ½ years at a rate of 6 ¾ % per annum. (3 marks)

$A = 25,500\left(1 + \frac{27}{400}\right)^{3.5}$ ✓

$= 32,050$

Interest $= 32050 - 25,500$ ✓
$= sh.\ 6550$ ✓

M1
M1
A1
03

9. Given that $\overrightarrow{OV} = \begin{pmatrix} -3 \\ 2 \\ 5 \end{pmatrix}$ and $\overrightarrow{OS} = \begin{pmatrix} 1 \\ -3 \\ -8 \end{pmatrix}$, express $\overrightarrow{VS}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ and find $|\overrightarrow{VS}|$ to 2dp. (3marks)

$\underset{\sim}{VS} = \begin{pmatrix} 1 \\ -3 \\ -8 \end{pmatrix} - \begin{pmatrix} -3 \\ 2 \\ 5 \end{pmatrix}$ ✓

$= 4\underset{\sim}{i} - 5\underset{\sim}{j} - 13\underset{\sim}{k}$ ✓

$|\underset{\sim}{VS}| = \sqrt{(4)^2 + (-5)^2 + (-13)^2}$ ✓

$= 14.49$ (2 d.p) ✓

B1
M1
A1
03

10. Make **y** the subject of the formula: (3 marks)

$V = \sqrt[3]{\frac{ax^2y}{w-y}}$

$\frac{V^3}{1} = \frac{ax^2y}{w-y}$ ✓

$v^3w - v^3y = ax^2y$

$ax^2y + v^3y = v^3w$

$y(ax^2+v^3) = v^3w$ ✓

$y = \frac{v^3w}{ax^2+v^3}$ ✓

M1 M1 A1 / 03

11. The sum of the first 4 terms of an arithmetic progression is 58 and the sum of the first 7 terms is 154. Find the first term and the common difference (3 marks)

$4a + 6d = 58$ }
$7a + 21d = 154$ } ✓

Solving the eqns. simultaneously

$d = 5$

$a = 7$

M1 — Equations

M1 — ✓ Attempt to solve

A1 — Both ✓

12. A Samsung led TV whose cash price is Kshs. 38,000 can be sold by two methods:
(i) 18 monthly installments of Ksh 2450.
(ii) Paying $\frac{1}{4}$ of the cash price as deposit and 17 monthly installments of Ksh1975.
Find the method of hire purchase terms that would be cheaper. (3 marks)

$M_1 = 18 \times 2450$
$= 44,100$

$M_2 = 9500 + (17 \times 1975)$
$= 43,075$

The 2nd method is cheaper

B1 B1 B1 / 03

13. Simplify, leaving your answer in surd form (3 marks)

$$\frac{-6}{\sqrt{13}+\sqrt{3}} - \frac{4}{\sqrt{3}-\sqrt{13}}$$

$$\frac{-6(\sqrt{3}-\sqrt{13})-4(\sqrt{3}+\sqrt{13})}{(\sqrt{3})^2-(\sqrt{13})^3}$$

$$\frac{-6\sqrt{3}+6\sqrt{13}-4\sqrt{3}-4\sqrt{13}}{3-13}$$

$$\frac{-10\sqrt{3}+2\sqrt{13}}{-10}$$

$$\sqrt{3}-\frac{1}{5}\sqrt{13}$$

M1 - Conjugate

M1 - Denominator Rationalized

A1 - Simplified

14. An arc subtends an angle of 0.9 radians at the centre of the circle. If the radius of the circle is 13cm, Find the length of the arc. (Take $\pi = 3.142$). (3 marks)

Length of arc $= \frac{\theta}{360} 2\pi r$

$1^\circ = 57.29^\circ$

$0.9^c = 0.9 \times 57.29$

$= 51.561^\circ$ B1

$\therefore L = \frac{51.561}{360} \times 2 \times 3.142 \times 13$ M1

$= 11.70$ cm A1

03

15. Determine the values of x given that $\begin{pmatrix} x & 1 \\ 2 & x+1 \end{pmatrix}$ is a singular matrix. (3 marks)

$x(x+1) - 2 = 0$ M1

$x^2 + x - 2 = 0$

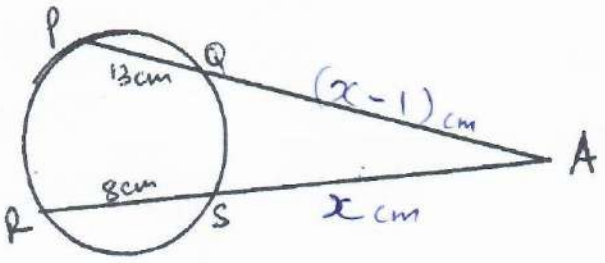
$x^2 + 2x - x - 2 = 0$

$x(x+2) - 1(x+2) = 0$ M1

$\therefore x = 1$ or $x = -2$ A1

03

16. In the figure below two chords PQ and RS intersect externally at A. PQ = 13cm, RS = 8cm. If AQ is 1cm shorter than AS. Find the length of AS. (3 marks)



Let $AS = x$ cm

$AP \cdot AQ = AR \cdot AS$

$(x+12)(x-1) = x(x+8)$ ——— M1

$x^2 - x + 12x - 12 = x^2 + 8x$

$11x - 8x = 12$ ——— M1

$3x = 12$

$x = 4$

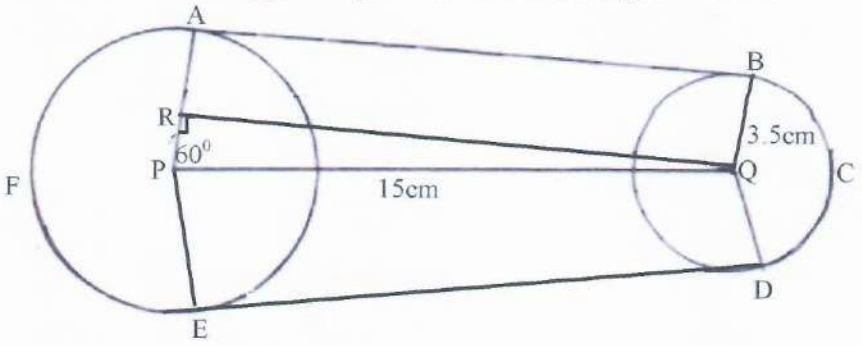
$\therefore AS = 4$ cm ——— A1

03

SECTION II: (50 MARKS)

Answer any five questions in the section in the spaces provided

17. The diagram below shows two fly wheels of centres **P** and **Q** connected with a belt **ABCDEF**. The radius of the smaller wheel is 3.5cm and the distance between **P** and **Q** is 15cm. Line **QR** is perpendicular to **AP**. Given that angle **APQ** = 60^0, calculate, using $\pi = 3.142$.



a) The length **AB** (2 marks)

$\sin 60 = \frac{RQ}{15}$ ✓

$RQ = 12.99$

But $RQ = AB$

$\therefore AB = 12.99\text{ cm}$ ✓ M1 A1

b) The radius of the bigger wheel. (2 marks)

$\cos 60 = \frac{PR}{15}$

$PR = 15 \times \cos 60$

$= 7.5$

Radius $= 3.5 + 7.5$ ✓

$= 11\text{ cm}$ ✓ M1 A1

c) The length of the belt to 1d.p (6 marks)

Length of Arc $AFE = \frac{240}{360} \times 2 \times 3.142 \times 11$ ✓ M1

$= 46.0827$ ✓ A1

Arc $BCD = \frac{120}{360} \times 2 \times 3.142 \times 3.5$ ✓ M1

$= 7.3313$ ✓ A1

Total $= 12.99 + 46.0827 + 12.99 + 7.3313$ ✓ M1

$= 73.394$

$\simeq 73.4$ (1 d.p) ✓ A1

10

18. The table below shows monthly income tax rates

Monthly taxable pay in K£	Rate of tax (shs. Per K£)
1 – 510	2
511 – 1020	3
1021 – 1532	4
1533 – 1948	5
1949 – 2564	6
Over 2564	7

A civil servant earns a monthly basic salary of sh. 38,000 and is provided with a house at a nominal rent of sh. 1,200 per month. Besides the basic salary, he is paid sh. 3,800 and sh. 2,700 as medical and commuter allowances per month respectively.

a) Given that taxable pay is the employee's income plus 15% of basic salary, less nominal rent, calculate the civil servants taxable income in K.£ (2 marks)

$T.I = \left(\frac{115}{100} \times 38,000\right) + 3800 + 2700 - 1200$ M1

$= sh.\ 49,000$

$= K£\ 2450$ A1

b) Calculate the gross tax per month. (5 marks)

$2 \times 510 = sh.\ 1020$

$3 \times 510 = sh.\ 1530$

$4 \times 512 = sh.\ 2048$

$5 \times 416 = sh.\ 2080$

$6 \times 502 = sh.\ 3012$

Gross tax = sh. 9690

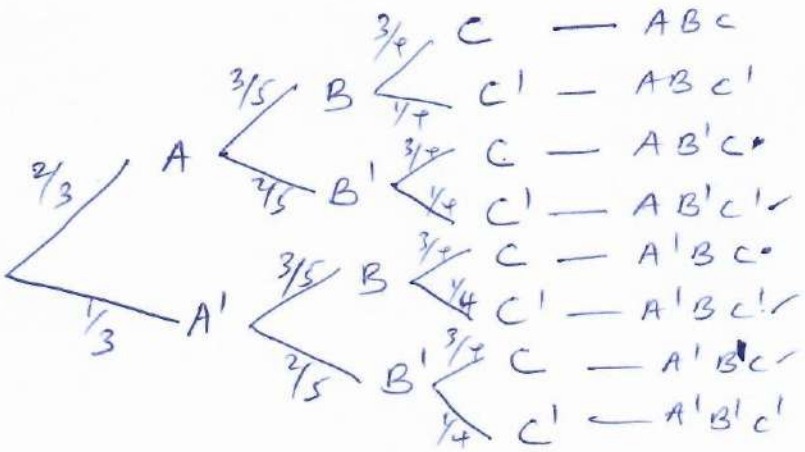
c) If the civil servant is entitled to a personal relief of sh. 1,820 per month, calculate the net monthly income. (3 marks)

Net income = $44500 - (7870 + 1200)$

$= sh.\ 35,430$

19. Three girls, Agnes, Beatrice and Christine, complete a crossword puzzle. The probability of getting it correct for each of them is $\frac{2}{3}$, $\frac{3}{5}$ and $\frac{3}{4}$ respectively.

a) Draw a tree diagram to represent the above information. (2 marks)



b) Using your tree diagram or otherwise, find the probability that:

i) None of the girls gets the puzzle correct. (2 marks)

$P(A'B'C') = \frac{1}{3} \times \frac{2}{5} \times \frac{1}{4}$

$= \frac{1}{30}$

ii) Exactly one girl gets it correct. (2 marks)

$\left(\frac{2}{3} \times \frac{2}{5} \times \frac{1}{4}\right) + \left(\frac{1}{3} \times \frac{3}{5} \times \frac{1}{4}\right) + \left(\frac{1}{3} \times \frac{2}{5} \times \frac{3}{4}\right)$

$= \frac{13}{60}$

iii) At least one girl gets it correct. (2 marks)

$1 - \left(\frac{1}{3} \times \frac{2}{5} \times \frac{1}{4}\right)$

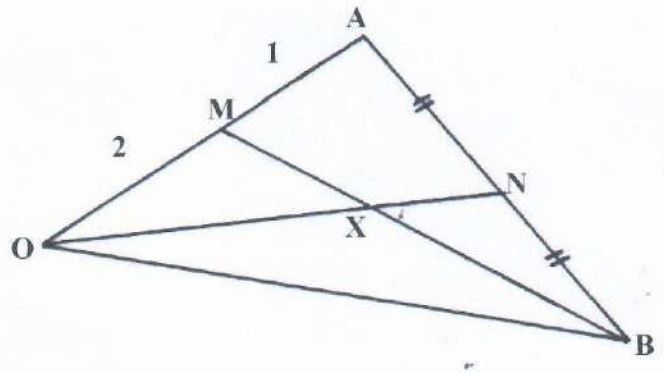
$= \frac{29}{30}$

iv) Exactly two girls get it correct if Christine has it correct. (2 marks)

$= \left(\frac{2}{3} \times \frac{2}{5} \times \frac{3}{4}\right) + \left(\frac{1}{3} \times \frac{3}{5} \times \frac{3}{4}\right)$

$= \frac{7}{20}$

20. The diagram below shows triangle **OAB** in which **N** is the midpoint of **AB** and **M** is a point of **OA** such that **OM**: **MA** = 2: 1 lines **ON** and **BM** meet at **X** such that **OX** = h**ON** and **MX** = k**MB**, where h and k are constants.



a) Given that $\underset{\sim}{OA} = \underset{\sim}{a}$ and $\underset{\sim}{OB} = \underset{\sim}{b}$, express **AB**, **ON** and **BM** in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$ (4 marks)

$\underset{\sim}{AB} = \underset{\sim}{b} - \underset{\sim}{a}$ ✓ B1

$\underset{\sim}{ON} = \underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$ ✓ B1

$= \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$ ✓ B1

$\underset{\sim}{BM} = -\underset{\sim}{b} + \frac{2}{3}\underset{\sim}{a}$ B1

or $\frac{2}{3}\underset{\sim}{a} - \underset{\sim}{b}$ ✓

b) By expressing **OX** in two different ways, determine the values of h and k. (6 marks)

$\underset{\sim}{OX} = h(\frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b})$

$= \frac{1}{2}h\underset{\sim}{a} + \frac{1}{2}h\underset{\sim}{b}$ --- (i) ✓

$\underset{\sim}{OX} = \frac{2}{3}\underset{\sim}{a} + k(\underset{\sim}{b} - \frac{2}{3}\underset{\sim}{a})$ ✓

$= \frac{2}{3}\underset{\sim}{a} + k\underset{\sim}{b} - \frac{2}{3}k\underset{\sim}{a}$ --- (ii)

$\frac{1}{2}h\underset{\sim}{a} + \frac{1}{2}h\underset{\sim}{b} = \frac{2}{3}\underset{\sim}{a} + k\underset{\sim}{b} - \frac{2}{3}k\underset{\sim}{a}$ ✓

$\frac{1}{2}h\underset{\sim}{a} = \frac{2}{3}\underset{\sim}{a} - \frac{2}{3}k\underset{\sim}{a}$

$\frac{1}{2}h = \frac{2}{3} - \frac{2}{3}k$ ←

$\frac{1}{2}h\underset{\sim}{b} = k\underset{\sim}{b}$ ✓

$k = \frac{1}{2}h$ ←

$\frac{1}{2}h = \frac{2}{3} - \frac{2}{3}(\frac{1}{2}h)$ B1

$\frac{1}{2}h = \frac{2}{3} - \frac{1}{3}h$ B1

$\frac{5}{6}h = \frac{2}{3}$ ✓

$h = 4/5$ M1

$k = 2/5$ M1 A1

B1 (Both)

10

21. In a certain term, a school did an order of some bags of maize and beans.
The school paid shs. 46,500 for 9 bags of maize and 3 bags of beans delivered in the first week and shs. 71,000 for 6 bags of maize and 7 bags of beans delivered in the second week.

i. Form two equations to represent the information above. (2 marks)

$9m + 3b = 46,500$ ✓ B1

$6m + 7b = 71,000$ ✓ B1

ii. Use matrix method to determine the cost of each bag of maize and each bag of beans. (5 marks)

$$\begin{pmatrix} 9 & 3 \\ 6 & 7 \end{pmatrix}\begin{pmatrix} m \\ b \end{pmatrix} = \begin{pmatrix} 46,500 \\ 71,000 \end{pmatrix}$$ ✓ M1

$$\frac{1}{45}\begin{pmatrix} 7 & -3 \\ -6 & 9 \end{pmatrix}\begin{pmatrix} 9 & 3 \\ 6 & 7 \end{pmatrix}\begin{pmatrix} m \\ b \end{pmatrix} = \frac{1}{45}\begin{pmatrix} 7 & -3 \\ -6 & 9 \end{pmatrix}\begin{pmatrix} 46,500 \\ 71,000 \end{pmatrix}$$ ✓ M1

$$\begin{pmatrix} m \\ b \end{pmatrix} = \begin{pmatrix} 2500 \\ 8000 \end{pmatrix}$$ ✓ A1

$m = Sh.\ 2500, \quad b = Sh.\ 8000$ B1

Each bag of Maize = Sh. 2500 ✓ B1

Each bag of beans = Sh. 8000 ✓

iii. In the following term, the supplier increased the price of maize per bag by 11% while the cost of beans per bag was decreased in the ratio 8:9. The school bought 17 bags of maize and 15 bags of beans. How much did the school pay to the supplier? (3 marks)

Price of maize per bag $= \frac{111}{100} \times 2500$

$= Sh.\ 2775$ ←

Price of beans $= \frac{8}{9} \times 8000$ ← M1

$= Sh.\ 7111.11$

Amount $= (17 \times 2775) + (15 \times 7111.11)$ M1

$= Sh.\ 153,841.65$ A1

10

22. (a) The first term of an Arithmetic Progression (AP) is 2. The sum of the first 8 terms of the AP is 156.

(i) Find the common difference of the AP (2 marks)

$\frac{8}{2}\{2\times 2 + 7d\} = 156$ M1

$28d = 140$

$d = 5$ A1

(ii) Given that the sum of the first n terms of the AP is 416, find n. (3 marks)

$\frac{n}{2}\{4 + (n-1)5\} = 416$ ✓

$5n^2 - n = 832$

$5n^2 - n - 832 = 0$

$n = \frac{+1 \pm \sqrt{(-1)^2 - (4\times 5\times -832}}{10}$ ✓

$n = \frac{+1 \pm 129}{10}$ M1

$= \frac{130}{10}$ M1

$= 13$ ✓ A1

(a) The 3rd, 5th and 8th terms of another AP form the first three terms of a Geometric progression (GP). If the common difference of the AP is 3, find:-

(i) The first term of the Gp. (4 marks)

$a+2d,\ a+4d,\ a+7d$ ✓

$\frac{a+4d}{a+2d} = \frac{a+7d}{a+4d}$ ✓

$(a+12)(a+12) = (a+6)(a+21)$

$a^2 + 24a + 144 = a^2 + 27a + 126$

$3a = 18$

$a = 6$ ✓

first term of GP B1

$= 6 + 6$ M1

$= 12$ ✓ A1

B1

(ii) The sum of the first 9 terms of the GP, to 4 significant figures. (2 marks)

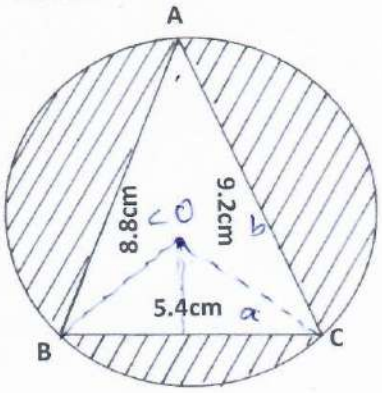
$r = 3/2$

$S_9 = \frac{12((1.5)^9 - 1)}{1.5 - 1}$ ✓

$= 74.89$ (4 s.f) ✓ M1 A1

10

23. The figure below shows a circle centre O, touching the vertices A, B and C of a triangle. AB=8.8, BC=5.4cm and AC=9.2cm.



Calculate to two decimal places;

(a) The value of angle A. (2 marks)

$a^2 = b^2 + c^2 - 2bc\cos A$ ✓

$5.4^2 = 9.2^2 + 8.8^2 - 161.92\cos A$

$\cos A = 0.8209$

$\angle A = 34.83^\circ$ (2 d.p) ✓ M1 A1

(b) The area of triangle ABC (2 marks)

$A = \frac{1}{2} \times 8.8 \times 9.2 \times \sin 34.83$ M1

$= 23.12\ cm^2$ A1

(c) The length of OC (2 marks)

$\sin 34.83 = \frac{2.7}{OC}$ ✓

$OC = 4.73\ cm$ ✓

ALT

$\frac{5.4}{\sin 34.83} = 2R$ ✓

$R = 4.73$ ✓

M1 A1

(d) The total shaded area (2 marks)

$(3.142 \times 4.7274^2) - (23.12)$ ✓

$= 47.10$ ✓

M1 A1

(e) The value of the largest angle of the triangle (2 marks)

$\frac{5.4}{\sin 34.83} = \frac{9.2}{\sin B}$

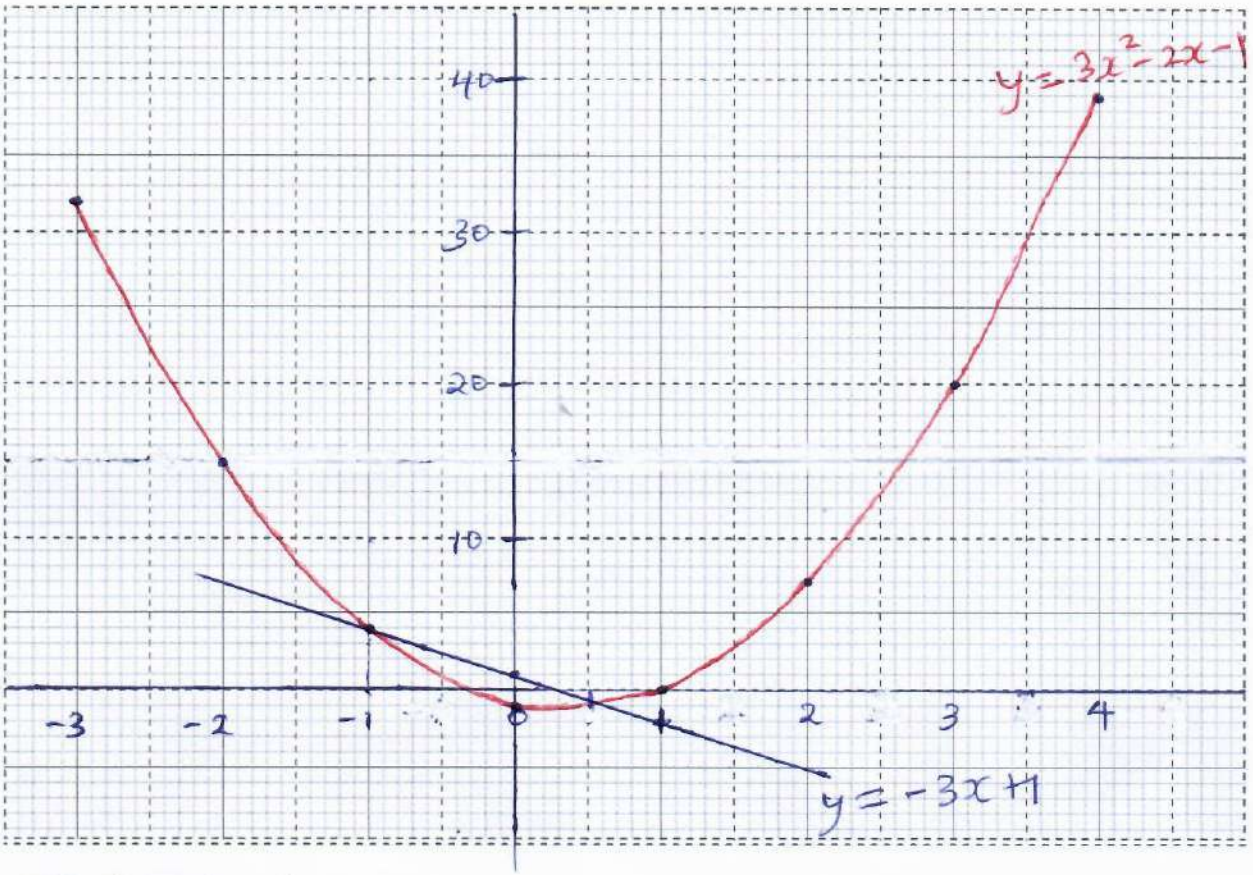
$\angle B = 76.67$

24. (a) Complete the table below for the function $y = 3x^2 - 2x - 1$ for -3 ≤x ≤4 (3 marks)

X	-3	-2	-1	0	1	2	3	4
$3x^2$	27	12	3	0	3	12	27	48
$-2x$	6	4	2	0	-2	-4	-6	-8
-1	-1	-1	-1	-1	-1	-1	-1	-1
y	32	15	4	-1	0	7	20	39

B3

(b)Draw the graph of $y = 3x^2 - 2x - 1$ (3 marks)



S1
P1
C1
L1

(c)Use the graph to solve;

(i) $3x^2 - 2x - 1 = 0$ (1 mark)

$x = -0.4$ or 1 B1

(ii) $3x^2 - 5x - 2 = 0$ (3 marks)

$y = -3x + 1$ B1

$x = -1$ or 0.5 B1