Name..MARKING SCHEME..... ADM Number:.....

School:..... Candidate's Signature.....

121/2
Mathematics Alt.A
FORM THREE.
FEBRUARY 2022.
2 ½ Hours.

URANGA MATHEMATICS ASSOCIATION-2021.

Kenya Certificate of Secondary Education

MATHEMATICS
121/2
FORM THREE
TIME: 2 ½ HOURS

INSTRUCTIONS TO CANDIDATES:

- Write your name, school, admission number and sign in the spaces provided above.
- This paper contains **TWO** sections: Section **I** and Section **II**.
- Answer **ALL** the questions in Section **I** and **FIVE** questions from section **II**.
- All answers and working **MUST** be written on the question paper in the spaces provided below each question.
- Marks may be given for correct working even if the answer is wrong.
- Non-programmable silent electronic calculators and KNEC Mathematical tables may be used, except where stated otherwise.

FOR EXAMINERS USE ONLY

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

SECTION II

17	18	19	20	21	22	23	24	Total

Grand Total

This paper consists of 15 printed pages. Candidates should check to ascertain that all pages are printed as indicated and that no questions are missing.

SECTION I (50 MARKS)

Answer ALL the questions in this section in the spaces provided.

1. The length and breadth of a rectangular paper were measured to be the nearest centimeter and found to be 20cm and 15 cm respectively. Find the percentage error in its perimeter leaving your answer to 4 significant figures. (3 marks)

Actual $P = 2(20+15)$
$= 70\text{cm}$

Min. $P = 2(19.5+14.5)$
$= 68\text{cm}$

Max. $P = 2(20.5+15.5)$
$= 72\text{cm}$

A.E $= \frac{1}{2} \times 4$ ✓
$= 2$

% error $= \frac{2}{70} \times 100$ ✓
$= 2.857$ ✓

M1
M1
A1
03

2. Simplify the following surds leaving your answer in the form $a + b\sqrt{c}$

$$\frac{\sqrt{5}}{2\sqrt{2}-\sqrt{5}} + \frac{\sqrt{2}}{2\sqrt{2}+\sqrt{5}}$$

(3 marks)

$$\frac{\sqrt{5}(2\sqrt{2}+\sqrt{5}) + \sqrt{2}(2\sqrt{2}-\sqrt{5})}{(2\sqrt{2}-\sqrt{5})(2\sqrt{2}+\sqrt{5})}$$

$N \Rightarrow 2\sqrt{10} + 5 + 4 - \sqrt{10}$
$= 9 + \sqrt{10}$

$D \Rightarrow 8 - 5 = 3$

$N/D = \frac{9+\sqrt{10}}{3}$ ✓
$= 3 + \frac{1}{3}\sqrt{10}$ ✓

M1 (Num)
M1 (Den.)
A1
03

3. Make Q the subject of the formula (3 marks)

$$T = P\sqrt{\frac{Q}{Q-1}}$$

$T^2 = P^2\left(\frac{Q}{Q-1}\right)$ ✓

$\frac{T^2}{1} = \frac{P^2 Q}{Q-1}$

$T^2(Q-1) = P^2 Q$

$T^2 Q - T^2 = P^2 Q$

$T^2 Q - P^2 Q = T^2$

$Q(T^2 - P^2) = T^2$ ✓

$Q = \frac{T^2}{T^2 - P^2}$ ✓

M1 (Squaring)
M1 (Factoring Q)
A1
03

4. Solve for x in $3\log_3 x + 4 = \log_3 24$ (3 marks)

$\log_3 x^3 + \log_3 81 = \log_3 24$ ✓

$\log_3 (81x^3) = \log_3 24$

$81x^3 = 24$

$x^3 = \frac{8}{27}$

$x = \sqrt[3]{\frac{8}{27}}$ ✓

$= \frac{2}{3}$ ✓

M1 M1 A1 03

5. (a) Expand $(2 + 2y)^5$. (1 mark)

2^5	2^4	2^3	2^2	2^1	2^0
$(2y)^0$	$(2y)^1$	$(2y)^2$	$(2y)^3$	$(2y)^4$	$(2y)^5$
1	5	10	10	5	1

$32 + 160y + 320y^2 + 320y^3 + 160y^4 + 32y^5$ B1

(b) Hence find the value of $(2.02)^5$, correct to 4 decimal places. (2 marks)

$2 + 2y = 2.02$

$2y = 0.2$

$y = 0.1$

$= 32 + 160(0.1) + 320(0.1)^2 + 320(0.1)^3 + 160(0.1)^4 + 32(0.1)^5$ ✓

$= 51.5363$ ✓

M1 A1 03

6. The fourth term of a geometrical progression is 40. The 7th term of the GP is 320. Determine the first term and the common ratio. (3 marks)

4^{th} term $= ar^3$

7^{th} term $= ar^6$

$\frac{ar^6}{ar^3} = \frac{320}{40}$ ✓

$r^3 = 8$

$r = \sqrt[3]{8}$

$= 2$ ✓

$a(2)^3 = 40$

$8a = 40$

$a = 5$ ✓

∴ First term $= 5$

Common Ratio $= 2$

7. If $\mathbf{a} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$; $\mathbf{b} = -3\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{c} = 11\mathbf{j} + 8\mathbf{k}$. (3marks)
Find the magnitude of the vector

$2\underset{\sim}{a} - \underset{\sim}{b} + \underset{\sim}{c}$

$= 2\begin{pmatrix}3\\1\\-2\end{pmatrix} - 1\begin{pmatrix}-3\\3\\4\end{pmatrix} + \begin{pmatrix}0\\11\\8\end{pmatrix}$

$= \begin{pmatrix}6\\2\\-4\end{pmatrix} - \begin{pmatrix}-3\\3\\4\end{pmatrix} + \begin{pmatrix}0\\11\\8\end{pmatrix}$

$= \begin{pmatrix}9\\10\\0\end{pmatrix}$ ✓

$|2\underset{\sim}{a} - \underset{\sim}{b} + \underset{\sim}{c}| = \sqrt{(9)^2 + (10)^2 + (0)^2}$ ✓

$= 13.45$ ✓

8. Solve the simultaneous equations. (4marks)

$2x + y = 5$

$2x^2 - 3y^2 = 5$

$2x + y = 5$

$y = 5 - 2x$

$2x^2 - 3(5-2x)^2 = 5$ ✓

$2x^2 - 3(25 - 20x + 4x^2) = 5$

$-10x^2 + 60x - 80 = 0$

$x^2 - 4x - 2x + 8 = 0$ ✓

$x = 2$ or $x = 4$ ✓

When $x = 2$

$y = 5 - 4$

$= 1$

When $x = 4$

$y = 5 - 8$

$= -3$

$x = 2, y = 1$

$x = 4, y = -3$ } ✓

9. Determine how long it would take for sh. 300,000 at 10% p.a. compounded semi-annually to accumulate to sh. 655,000. (3 marks)

$655,000 = 300,000\left(1 + \frac{5}{100}\right)^{2n}$

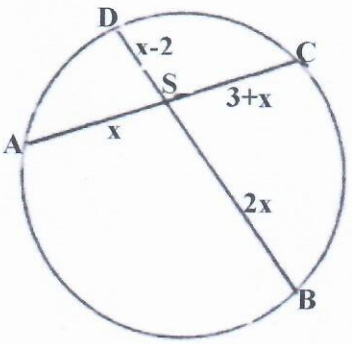
$2.18333 = (1.05)^{2n}$

$2n \log 1.05 = \log 2.18333$

$2n = 16$

$n = 8$

10. Two chords of the circle **ABCD**; **AC** and **BD** intersect at **S**. **AS** = x, **SC** = (3+ x), BS = 2x and **SD**= (x- 2). Calculate the value of **x.** (3 marks)



$AS \cdot SC = BS \cdot SD$

$x(3+x) = 2x(x-2)$

$2x^2 - 4x = 3x + x^2$

$x^2 - 7x = 0$

$x(x-7) = 0$

$\therefore x = 7$

11. A relation connecting three variables **R, C** and **T** is such that **R** varies directly as the square of **C** and inversely as **T**. When **R**= 30, C= 6 and **T** = 2.4, find the equation connecting **R**, **C** and **T**. (3 marks)

$R \propto \frac{C^2}{T}$

$R = \frac{kC^2}{T}$

$\frac{30}{1} = \frac{k \cdot 6^2}{2.4}$ ✓

$36k = 72$

$k = 2$ ✓

$\therefore R = \frac{2C^2}{T}$ ✓

12. Mr. Kowino a property ~~developer~~ developer, borrowed Ksh. 5.4 million from a bank to buy land. He was required to repay the loan with a simple interest for a period of 72 months. The monthly repayment was Ksh. 129 000. Calculate:

(a) Total interest paid to the bank. (2 marks)

Total Amount = $72 \times 129{,}000$

$= 9{,}288{,}000$ ✓

Interest = $9{,}288{,}000 - 5{,}400{,}000$

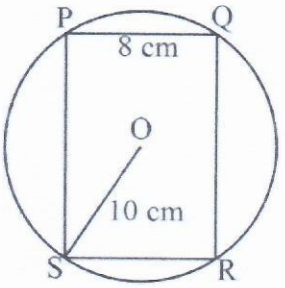
$=$ Sh. $3{,}888{,}000$ ✓

(b) The rate per annum of the simple interest. (2 marks)

$\frac{5{,}400{,}000 \times R \times 6}{100} = 3{,}888{,}000$ ✓

$R = 12\%$ ✓

13. The figure below shows a circle centre O and radius 10 cm with a rectangle PQRS inscribed in it. Chord PQ = 8 cm.



Find to one decimal place:

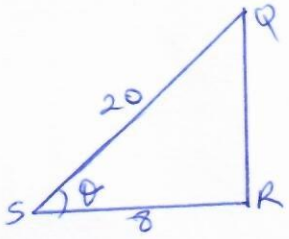
(c) The length of chord PS. (2 marks)

$PS = \sqrt{20^2 - 8^2}$ ✓

$= 18.3$ cm ✓

M1 A1 B1

(d) Angle subtended by chord QR at the centre of the circle. (2 marks)



$\cos\theta = 8/20$

$\theta = 66.42°$

$\therefore \angle$ at the centre $= 132.8°$

M1 A1

14. In a month a school buys 60 bags of maize, 32 bags of beans, 64 bags of rice and 16 bags of sugar. The prices per bag of each commodity are Ksh 3, 200, Ksh 5, 200, Ksh 4, 800 and Ksh 4, 500 respectively. Using a 1x 4 matrix to represent the number of bags bought by the school, and a 4 x1 matrix to represent the cost of the above commodities, find the amount of money the school spends in a month. (3 marks)

No. of bags $= (60 \quad 32 \quad 64 \quad 16)$ ✓

$\text{Cost} = \begin{pmatrix} 3,200 \\ 5,200 \\ 4,800 \\ 4,500 \end{pmatrix}$

M1

M1

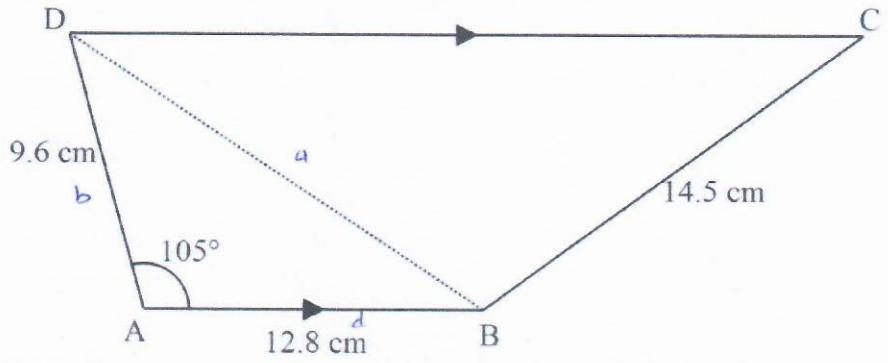
$\text{Amount} = (60 \quad 32 \quad 64 \quad 16)\begin{pmatrix} 3200 \\ 5200 \\ 4800 \\ 4500 \end{pmatrix}$ ✓

A1

$= \text{Sh. } 737,600$ 6 ✓

03

15. In the trapezium ABCD below, AB = 12.8 cm, BC = 14.5 cm and AD = 9.6 cm. AB is parallel to DC and angle DAB = 105°.



Calculate, correct to one decimal place the length of BD. (3 marks)

$a^2 = b^2 + d^2 - 2bd\cos A$

$= 9.6^2 + 12.8^2 - 2 \times 9.6 \times 12.8 \times \cos 105°$ ✓ M1

$= 256 + 63.6074$

$= 319.6074$

$\therefore a = \sqrt{319.6074}$ ✓ M1

$= 17.9$ cm ✓ A1

03

16. A point P divides line **AB** in the ratio 7:-5 where A and B are the points A(2,5,4) and B(-4,7,2). Find the co-ordinates of P. (3 marks)

m n

$= \frac{-5}{2}\begin{pmatrix} 2 \\ 5 \\ 4 \end{pmatrix} + \frac{7}{2}\begin{pmatrix} -4 \\ 7 \\ 2 \end{pmatrix}$ ✓ M1

$= \begin{pmatrix} -19 \\ 12 \\ -3 \end{pmatrix}$ ✓ M1

$\therefore$ Coordinates of P $(-19, 12, -3)$ ✓ A1

03

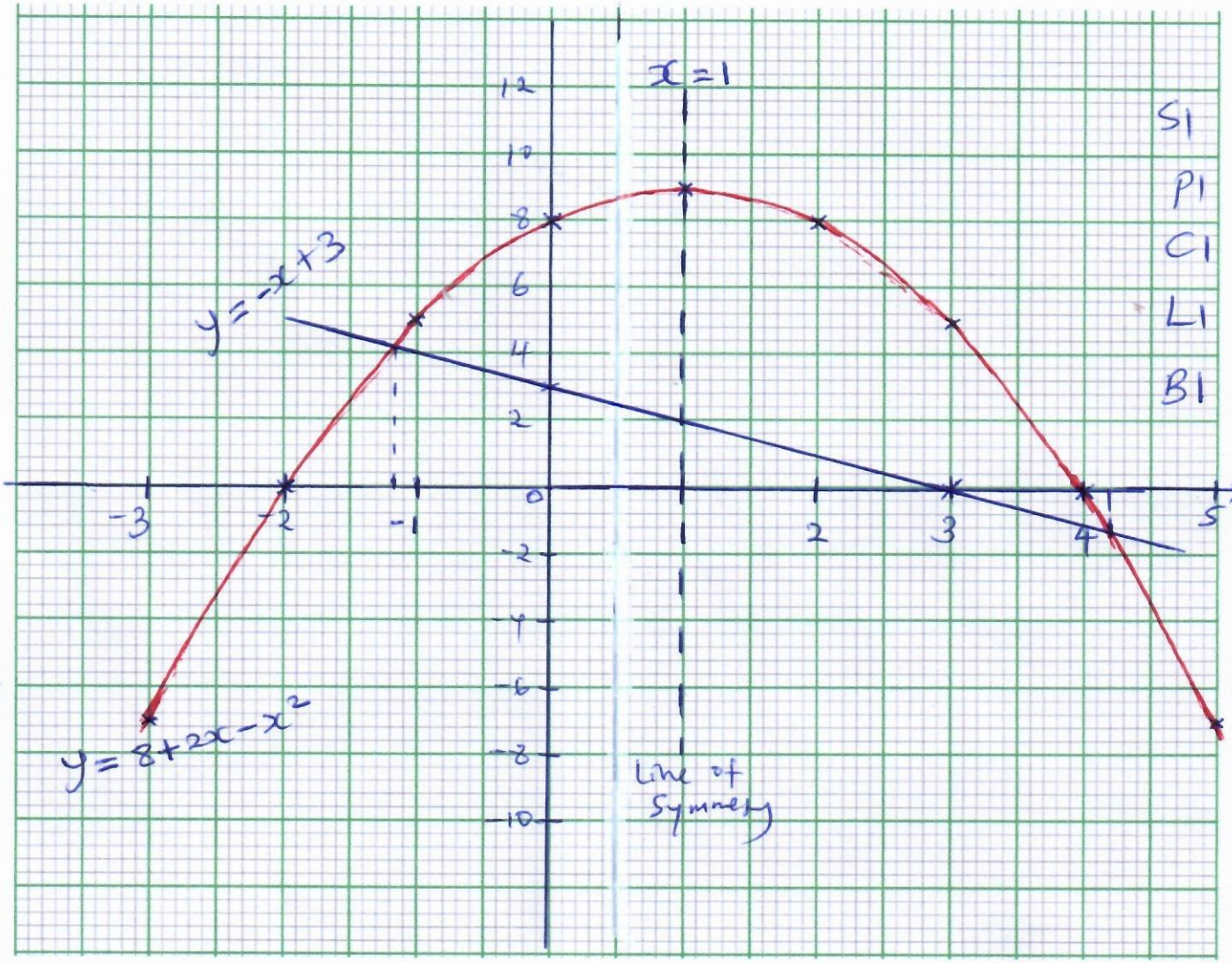
SECTION II (50 Marks)

Answer ANY FIVE Questions in the spaces provided.

17. .(a) Complete the table below for $y = 8 + 2x - x^2$ (2 marks)

x	-3	-2	-1	0	1	2	3	4	5
Y	-7	0	5	8	9	8	5	0	-7

b) On the grid provided, draw the graph of $y = 8 + 2x - x^2$ for the range $-3 \leq x \leq 5$. (3 marks)



c) i) Use your graph to solve the equation $-x^2 + 3x + 5 = 0$ (3 marks)

$y = -x + 3$ B1

$x = -1.15$ or $x = 4.2$ B1

ii) State the equation of the line of symmetry for the curve (2 marks)

$y = 8 + 2x - x^2$

$x = 1$ B1

10

18. Nancy bought a car for Ksh. 800,000. The value of the car depreciated at the rate of 10% p.a. for 3 years

a) Determine the value of the car at the end of 3 years. (3 marks)

$A = P(1 - r/100)^n$ M1

$= 800,000(1 - \frac{10}{100})^3$ M1

$= Sh.\ 583,200$ A1

b) Nancy sold the car at the value calculated in (a) above and used the money to buy a piece of land. The value of the land appreciated at the rate of 15% p.a. for the first year and then at 12% p.a. in the second and third year.

i) Calculate the value of the land at the end of the first year. (2 marks)

$A = 583,200(1 + 15/100)^1$ M1

$= 670,680$ A1

ii) Calculate the value of the land, to the nearest shillings at the end of the third year. (2 marks)

$A = 670,680(1 + \frac{12}{100})^2$ M1

$= 841,300.992$

$\simeq 841,301$ A1

iii) Determine, to 3 significant figures, the percentage gain in Nancy's land investment at the end of the 3 years. (3 marks)

$Gain = 841,301 - 583,200$ ✓ M1

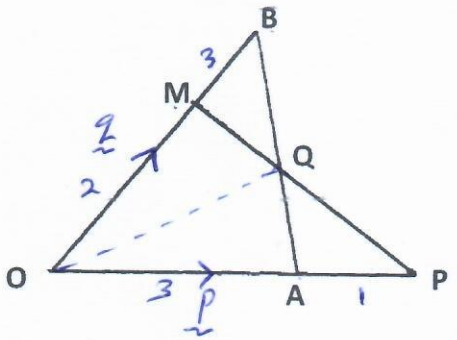
$= Sh.\ 258,101$

$\%\ Gain = \frac{258101}{583,200} \times 100$ ✓ M1

$= 44.3\ \%$ ✓ A1

10

19. In the figure below, **OA**= **p** and **OB** = **q**. **OM: MB= 2:3** and **OP** = $\frac{4}{3}$ **OA**



a) Express **AB, OM** and **MP** in terms of **p** and **q** (3 marks)

$\underset{\sim}{AB} = \underset{\sim}{AO} + \underset{\sim}{OB}$

$= -\underset{\sim}{p} + \underset{\sim}{q}$

$= \underset{\sim}{q} - \underset{\sim}{p}$ ✓ B1

$\underset{\sim}{OM} = \frac{2}{5}\underset{\sim}{q}$ ✓ B1

$\underset{\sim}{MP} = -\frac{2}{5}\underset{\sim}{q} + \frac{4}{3}\underset{\sim}{p}$

$= \frac{4}{3}\underset{\sim}{p} - \frac{2}{5}\underset{\sim}{q}$ ✓ B1

b) Given that **AQ** = k**AB**, express **OQ** in terms of **p, q** and **k** (1 mark)

$\underset{\sim}{OQ} = \underset{\sim}{p} + k(\underset{\sim}{q} - \underset{\sim}{p})$

$= \underset{\sim}{p} + k\underset{\sim}{q} - k\underset{\sim}{p}$ - - - - - - - - B1

c) Given that **MQ** = h**MP**, express **OQ** in terms of **p, q** and **h** (1 mark)

$\underset{\sim}{OQ} = \frac{2}{5}\underset{\sim}{q} + h(\frac{4}{3}\underset{\sim}{p} - \frac{2}{5}\underset{\sim}{q})$ - - - - - B1

d) Using your answers to parts (b) and (c) or otherwise, find the values of **h** and **k**. (5 marks)

$\frac{2}{5}\underset{\sim}{q} + \frac{4}{3}\underset{\sim}{p}h - \frac{2}{5}\underset{\sim}{q}h = \underset{\sim}{p} + k\underset{\sim}{q} - k\underset{\sim}{p}$ ✓

$\frac{2}{5}\underset{\sim}{q} - \frac{2}{5}\underset{\sim}{q}h = k\underset{\sim}{q}$

$\frac{2}{5} - \frac{2}{5}h = k$ - - (i)

$\frac{4}{3}\underset{\sim}{p}h = \underset{\sim}{p} - k\underset{\sim}{p}$

$\frac{4}{3}h = 1 - k$ - - (ii) ✓

$\frac{4}{3}h = \frac{3}{5} + \frac{2}{5}h$

$h = \frac{9}{14}$ ✓

$k = \frac{2}{5} - \frac{2}{5}(\frac{9}{14})$ ✓ M1

$= \frac{1}{7}$ ✓ m1 m1 A1

10

20. An arithmetic progression is such that the first term is -5, the last is 135 and the sum of the progression is 975. Calculate:

(i) The number of terms in the series (3 marks)

$a = -5$

$l = 135$

$\frac{n}{2}\{-5 + 135\} = 975$ ✓ M1

$\frac{130n}{2} = 975$ ✓ M1

$n = 15$ ✓ A1

(ii) The common difference of the progression (2 marks)

$\frac{15}{2}\{-10 + (14d)\} = 975$ ✓ M1

$14d - 10 = 130$

$14d = 140$ A1

$d = 10$ ✓

(b) The sum of the first three terms of a geometric progression is 27 and first term is 36. Determine the common ration and the value of the fourth term (5 marks)

$a + ar + ar^2 = 27$ ✓

$36 + 36r + 36r^2 = 27$ M1

$4r^2 + 4r + 1 = 0$ M1

$(2r + 1)(2r + 1) = 0$ ✓

$2r = -1$ M1

$r = -\frac{1}{2}$ ✓ M1

4th term $= ar^3$ A1

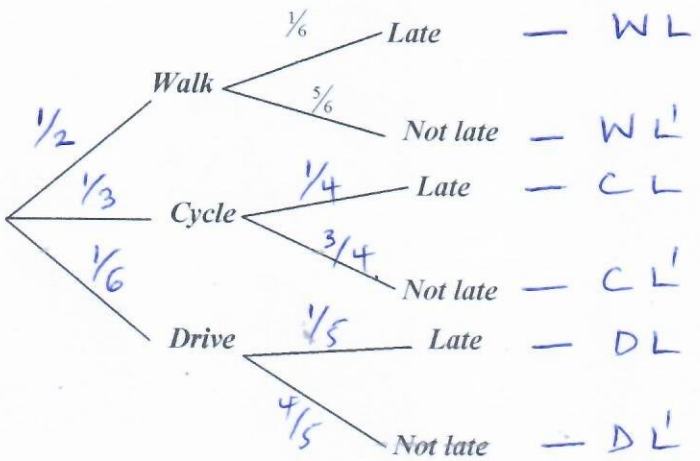
$= 36\left(-\frac{1}{2}\right)^3$ ✓

$= -4.5$ ✓

10

21. Each morning a teacher either walks, cycles or drives to school. The probabilities of each of these events are $\frac{1}{2}, \frac{1}{3}, and\ \frac{1}{6}$ respectively. If she walks, the probability that she will arrive late is $\frac{1}{6}$. If she cycles, the probability that she will arrive late is $\frac{1}{4}$. If she drives, the probability that she will arrive late is. $\frac{1}{5}$.

a) Complete the tree diagram below, filling in the probabilities. (3 marks)



B1

B1

B1

b) Find the probability that:

i) She cycles and is late (2 marks)

$P(CL) = \frac{1}{3} \times \frac{1}{4}$ M1

$= \frac{1}{12}$ A1

ii) She does not walk and is not late (2 marks)

$= \left(\frac{1}{3} \times \frac{3}{4}\right) + \left(\frac{1}{6} \times \frac{4}{5}\right)$ M1

$= \frac{23}{60}$ A1

iii) She is late. (3 marks)

$= \left(\frac{1}{2} \times \frac{1}{6}\right) + \left(\frac{1}{3} \times \frac{1}{4}\right) + \left(\frac{1}{6} \times \frac{1}{5}\right)$ M1 M1

$= \frac{1}{5}$ A1

10

22. (a) A quantity Y varies partly as the square root of X and partly as the inverse of the square of X. When X = 4, Y = 9.5 and when X = 1, Y = – 3. Determine the equation connecting X and Y, hence find Y when X = 9. (4 marks)

$Y = m\sqrt{x} + \frac{n}{x^2}$ ✓

$9.5 = m\sqrt{4} + \frac{n}{4^2}$

$9.5 = 2m + \frac{n}{16}$ --(i)

$-3 = m\sqrt{1} + \frac{n}{1^2}$

$-3 = m + n$

$32m + n = 152$

$m + n = -3$ ✓

$31m = 155$

$m = 5$

$n = -8$

Equation ✓

$Y = 5\sqrt{x} - \frac{8}{x^2}$

When $x = 9$

$Y = 5\sqrt{9} - \frac{8}{81}$

$= 14\frac{73}{81}$ ✓

(b) Three quantities P, Q and R are such that P varies as the square of Q and inversely as the cube root of R.

(i) Given that P = 32 when Q = 8 and R = 125, find the value of P when Q = 20 and R = 512. (3 marks)

$P = \frac{KQ^2}{\sqrt[3]{R}}$

$32 = \frac{64K}{5}$

$K = 2.5$

$\therefore P = \frac{2.5Q^2}{\sqrt[3]{R}}$ ✓

When $Q = 20$, $R = 512$

$P = \frac{2.5(20)^2}{\sqrt[3]{512}}$ ✓ M1

$= 125$ ✓ M1

A1

(ii) If Q is increased by 10% and R decreased by 30%, find the percentage change in P. (3 marks)

$P_1 = \frac{(1.1Q)^2 K}{\sqrt[3]{0.7R}}$ M1

$P_1 = 1.3628\frac{KQ^2}{\sqrt[3]{R}}$ M1

$P_1 = 136.28\%$

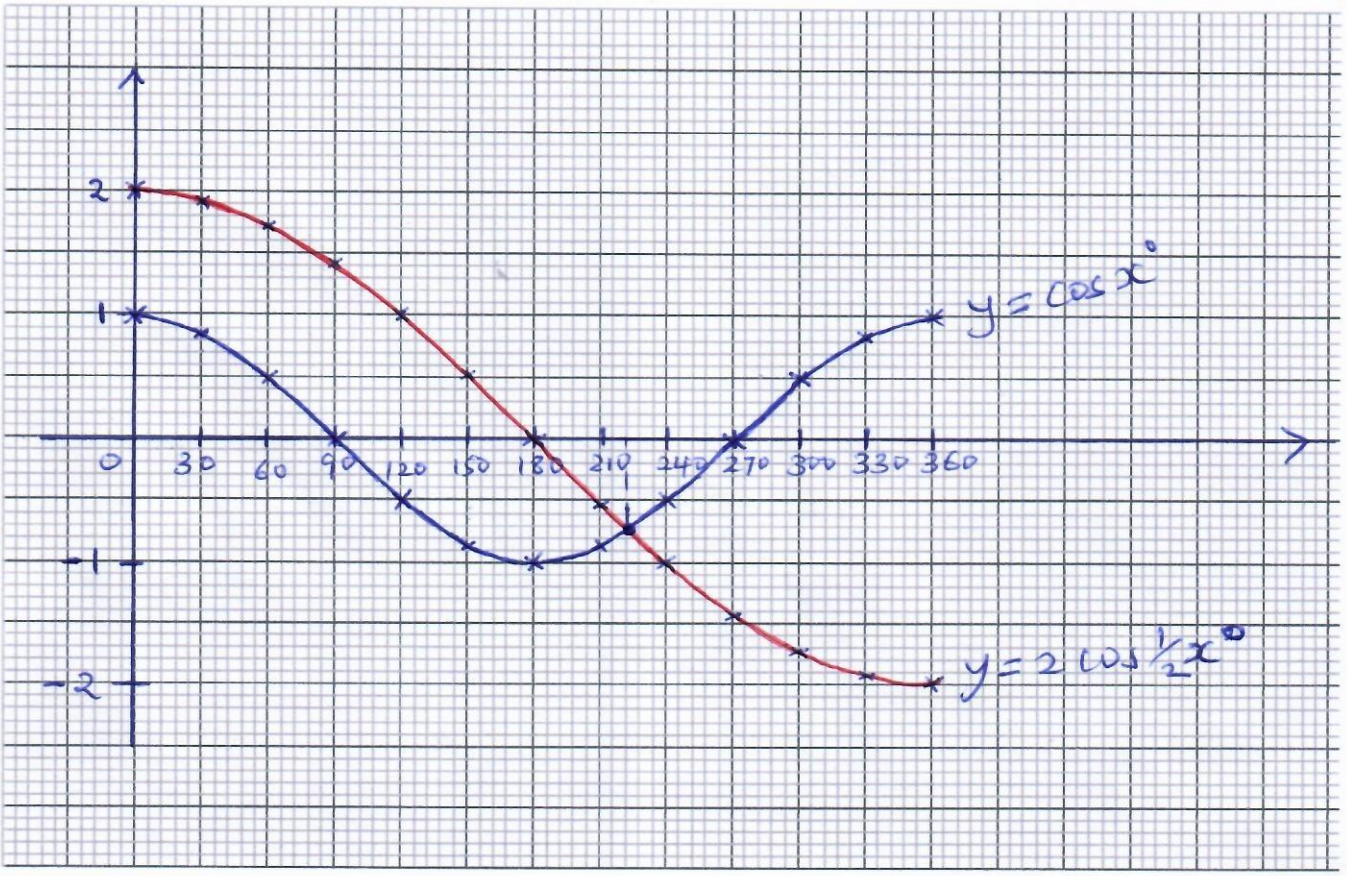
P has increased by 36.28 % A1

10

23. Complete the table below by filling in the blank spaces. (3 marks)

x^0	0^0	30^0	60^0	90^0	120^0	150^0	180^0	210^0	240	270^0	300	330	360^0
Cos x^0	1.00	0.87	0.50	0.00	−0.50	-0.87	−1.00	-0.87	−0.50	0	0.50	0.87	1.00
2 Cos ½ x	2.00	1.93	1.73	1.41	1.00	0.52	0.00	−0.52	−1.00	−1.41	−1.73	−1.93	−2.00

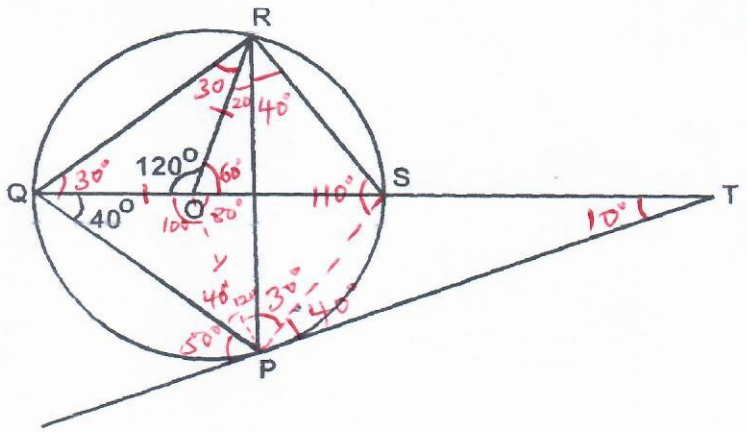
a) On the grid provided using a scale of 1cm to represent 30^0 on the horizontal axis and 2 cm to represent 1 unit on the vertical axis draw the graph of $y = \cos x^0$ and $y = 2\cos \frac{1}{2}x^0$ (5 marks)



b) Use your graph to solve the equation (2 marks)
$2 \cos \frac{1}{2}x - \cos x = 0$

$x = 222^\circ$

24. In the figure below **P**, **Q**, **R** and **S** are points on the circumference of a circle centre **O**. The points **T**, **S**, **O** and **Q** lie on a straight line, **PT** is a tangent to the circle at **P**.



Find the values of the following angles stating reasons in each case.

a) ∠**SRP** (2 marks)

40° (Angles subst. by the same chord (SP) are equal)

b) ∠**ORP** (3 marks)

20° (Angles in alternate segments are equal)

c) ∠**RPT** (3 marks)

70° (Angles on a straight line add upto 180°)

d) ∠**STP** (2 marks)

10° (Angles in a triangle add upto 180°)